Optical Activity in Small Molecules, Nonenantiomorphous Crystals, and Nematic Liquid Crystals
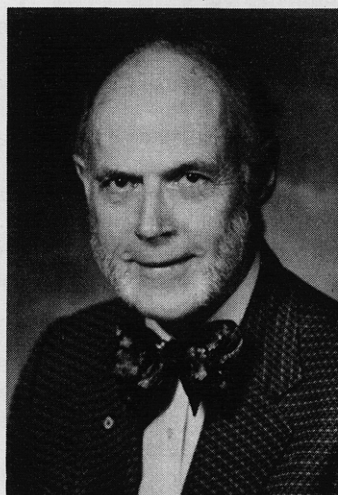
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Contents

I. Introduction

Although the necessary condition for optical activity for small molecules in the isotropic liquid phase has been known for more

Kenneth O'Loane, born in 1913 in Washington State, is a Research Associate, Research Laboratories, Eastman Kodak Co. He received his B.S. from St. Benedict's College (Kansas), M.S. from University of Washington, and Ph.D. with Professor E. Bright Wilson, Jr., at Harvard for work in the far-infrared. After becoming Associate Professor of Chemistry at the University of New Hampshire, he joined Eastman Kodak Co. in 1954. In 1959 he became a member of the Research Laboratories and, with Dr. R. L. Griffith, introduced nuclear magnetic resonance spectroscopy into these laboratories. Most recently he has been applying Fourier transform IR spectroscopy to polymer films. Besides optical activity his interests include polymer thermodynamics.

than 50 years,[1-3] there is still much confusion in both literature and textbooks. In spite of the availability of good American and English texts on symmetry and group theory, several new books appear each year referring to the basis of optical activity, or of optical rotatory dispersion, for small molecules in the isotropic liquid phase, as asymmetry. Pasteur, however, stated clearly and correctly about 130 years ago that it is *dissymétrie moléculaire—dissymmetry*.[4,5]

Further, there are misunderstandings on two aspects of optical activity in crystals. First, chemists usually do not know that the necessary conditions for optical activity in crystals are fundamentally different from that for small molecules. Second, there is often confusion between linear birefringence (ordinary double refraction), which can cause linearly, circularly, or even elliptically polarized light in optically inactive solids, such as calcite, and circular birefringence. It is circular birefringence (circular double refraction, allogyric double refraction) that causes the rotation of the plane of linearly polarized light in ordinary optical activity, as manifested in crystals of quartz or solutions of sugar. To understand ellipsometry, optical rotatory dispersion (ORD), which is simply optical activity as a function of wavelength, and circular dichroism (CD), one must be very clear on these points.

Section II attempts to lay to rest some 50–60 errors concerning optical activity which are endemic in the literature by reviewing the history of how these errors occurred. It provides a foundation for those who did not have symmetry point groups in their college chemistry courses to understand the basis of optical activity. The requisite information on symmetry properties of molecules is embedded in a historical matrix sufficient to

provide both interest and palatability. For further study, some texts on symmetry and group theory are recommended.

Although errors are not attributed by name to any authors within the past 40 years, explicit references to the errors of the pioneers in the field of optical activity are given. I believe it is no derogation of the first two Nobel prize winners, J. H. van't Hoff and Emil Fischer, to point out their mistakes. Rather, it gives a better feel for the way science progresses to see that the greater scientists, too, fall down wells, and get out of the trap either by themselves or with the help of others. With 20/20 hindsight, often obtained by standing on the shoulders of giants, all scientists see more clearly.

In section II the conditions for optical activity are always those for small molecules in the isotropic liquid phase. Optical rotation will be considered simply as the rotation of the plane of linearly polarized light by an isotropic material (neat liquid or solution). Only in section II.F, Potential Optical Activity, and in section III.E.2, Circular Birefringence and Circular Dichroism, is it explicitly mentioned that optical activity is physically related to absorption of energy, usually electronic energy.

Since the criterion for optical activity enunciated by Pasteur—dissymétrie moléculaire—is both correct and fundamental, it is recalled in section II.A.

In section II.B, a more precise specification of the root of dissymmetry, made possible by van't Hoff's first and second cases of optical activity, is discussed. The "second case of optical activity", which is the more fundamental one, is emphasized and illustrated with allene derivatives.

A historic error of Pope with respect to (4-methylcyclo-hexylidene)acetic acid is corrected in section II.C. The resolution of this error reveals an essential difference, not hitherto recognized, I believe, between derivatives of allene and those of 4-methylcyclohexylidenes and the corresponding spiranes.

Historically, attempts to refine still further the roots of dissymmetry in molecules were interwoven with efforts to improve the original Le Bel–van't Hoff definition of the asymmetric carbon atom. Recent ideas for refining the definition of an asymmetric carbon atom have been beclouded because of confusion in the literature as to whether van't Hoff thought asymmetric carbon atoms could exist in cyclic compounds. This confusion is cleared up in section II.D.

Enough historical background on the use of the terms "asymmetric atom" and "pseudoasymmetric atom" is given in section II.E for understanding the IUPAC rules for stereochemical nomenclature on these topics. "Diastereomeric atom" is suggested as preferable to "pseudoasymmetric atom".

In the second half of section II.E a contemporary difficulty which comes from confusing symmetric–asymmetric with nondissymmetric–dissymmetric (achiral–chiral) will be clarified by a table classifying molecular point groups in two different ways. Since this table is more expanded than the highly condensed tables given elsewhere, the fundamentally different questions involved in these classifications are more readily apparent.

The IUPAC committee for organic stereochemical nomenclature appears to have confused the condition for potential optical activity with the necessary and sufficient condition for it. Suggestions are made in section II.F.

The sufficient condition for optical activity in small molecules in the isotropic liquid phase, which has become somewhat obscured in the past 25 years, is stated explicitly in section II.G, and in section II.H the term "homomer" is proposed for a molecule or object which is superposable on its mirror image.

Although there are also a number of errors in current literature with respect to optical activity in crystals, the textbooks and literature of chemistry have been affected only slightly by them. In section III the emphasis is not on the origins and resolutions of a large number of errors but on the fact that, in crystals,
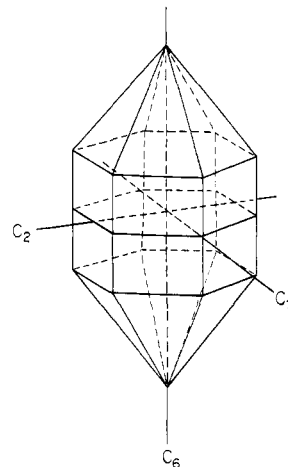

Figure 1. Holohedral hexagonal quartz crystal, point group D_{6h}.

enantiomorphism is not the sole root of optical activity.

The purpose is principally to call attention to the prediction of J. Willard Gibbs in 1882 that optical activity should be possible in two classes of nonenantiomorphous crystals, and to the observation of optical activity in members of both these classes in 1967 and 1968.

No attempt is made to give a fundamental explanation of this. Instead, the differences between linear and circular birefringence will be discussed briefly, so that those who have not had intermediate optics will be able to understand why it took so long to find crystals satisfying Gibbs's predictions.

A few references will be given to the components of the optical gyration tensor where these shed some light on the necessary conditions for optical activity in crystals.

Just as with nonenantiomorphous crystals, nematic liquid phases with C_s and C_{2v} symmetry may be optically active. The observation of optical activity in a case for which the symmetry of the liquid phase is C_s is discussed briefly in section IV.

II. Optical Activity in Small Molecules in the Isotropic Liquid Phase: J. H. van't Hoff

To understand van't Hoff's two cases of optical activity, we first consider the basic criterion for this phenomenon laid down by Pasteur in 1848.

A. Pasteur's Criterion for Optical Activity: Dissymétrie Moléculaire

The habit, i.e., the external morphology, of many α-quartz (low quartz) crystals is a symmetrical hexagonal prism, capped at both ends with a symmetrical hexagonal pyramid.[6] Such crystals (Figure 1) have the symmetry of point group D_{6h}. That is, they have a sixfold axis of symmetry along the optic axis, six twofold axes of symmetry perpendicular to it, and several planes of symmetry, as well as other elements of symmetry not of interest here.[6-8a]

Near the end of the eighteenth century the Abbé R. J. Haüy, a canon of the Cathedral of Notre Dame and Professor of Humanities at the University of Paris, noted that a small percentage of α-quartz crystals had little extra faces (hemihedral faces). Such crystals can exist as right- and left-handed forms, related as an object and its nonsuperposable mirror image (Figure 2), and are said to exhibit hemihedry. The unusual view in this figure has been chosen because it shows not only the enantiomorphous relation of these forms of quartz but also the degeneration of the sixfold axis into a threefold axis and the disappearance of all planes of symmetry. The threefold axis is also the optic axis.
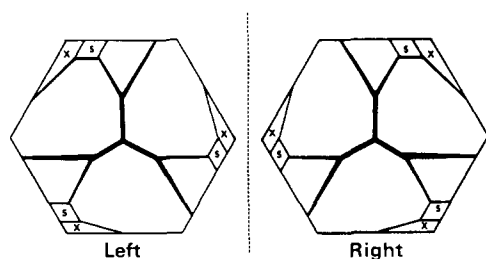
Figure 2. Left- and right-handed α-quartz crystals, point group D_3, viewed from the threefold axis (after F. Rinne,[8b] courtesy of E. P. Dutton and Co.; also after Klug and Alexander,[8c] courtesy of John Wiley and Sons, Inc.).

It is often said that the symmetry of such crystals has been destroyed—i.e., that they are asymmetric. This is incorrect. Although there are no longer any planes of symmetry, but only axes, the symmetry of the crystals is still quite high—point group D_3 (section II.B.2).

About 1812 the French physicist Biot discovered that there are two kinds of α-quartz, differing in optical activity.[9,10] When cut perpendicular to the *direction*[11] in the crystal called the optic axis, one kind of quartz rotates the plane of linearly polarized light to the left, the other to the right.

In 1820 the English astronomer Sir John Herschel showed that α-quartz crystals displaying right-handed hemihedry were dextrorotatory, and those with left-handed hemihedry were levorotatory.[12a] In α-quartz, then, morphological hand coincides with optical hand.[12b]

The discussion to this point may well have given the novice in crystallography the impression that only quartz crystals with hemihedral faces are optically active. However, potential optical activity is determined by the *structural* symmetry and not by *morphological* symmetry, the symmetry of the external habit.[12b] By structural symmetry both hemihedral and holohedral quartz crystals belong to symmetry point group D_3 and are optically active.

The morphological symmetry may be higher than the structural symmetry; this is true for α-quartz, in which the morphological symmetry may be D_{6h} (Figure 1), but the structural symmetry is always D_3 (Figure 7). The appearance of the hemihedral faces in some quartz crystals is an external manifestation that the true symmetry class of quartz is lower than would appear from the common morphology.[12c,d]

Pasteur was aware of Biot's findings, as well as those of Haüy and Herschel. He also knew that Biot had found that solutions of *d*-tartaric acid, where the crystal arrangement is destroyed, were dextrorotatory.

In his investigations of the tartaric acids and tartrates, both crystals and solutions,[13] Pasteur made the following discoveries: (1) Nineteen different dextrorotatory tartrate salts all showed right-handed hemihedry.[14,15] (2) The sodium ammonium tartrates formed from racemic acid[16] manifest the same kind of hemihedry previously found by Haüy in quartz crystals. (3) There is a correlation between the handedness of these tartrate crystals and the rotatory power of their aqueous solutions.[5,14] (4) A new kind of tartaric acid, the levorotatory, was isolated and identified. (5) The third isomer of tartaric acid, *meso*-tartaric acid, was also isolated and identified.

Further investigations led him to conclude that *nonsuperposability of object and mirror image* (*molecular dissymmetry*,[17] or, as we should say today, *chirality*) is the *necessary* condition for optical activity in both small molecules (in the isotropic liquid phase) and crystals. Although this conclusion, as we shall see in the next section, is incorrect for crystals, it is correct for small molecules. Not only has no exception to it ever been discovered, but the quantum mechanical bases for optical *inactivity* in those molecules which *are* superposable on their mirror images—the homomers as I shall suggest that they be called (section II.

H)—have been elucidated.[18-22]

To describe this property of nonsuperposability of object and mirror image, Pasteur in 1848 invented the phrase "dissymétrie moléculaire".[5] Commenting on the etymology of this new French term, Sir William J. Pope (1870–1939), noted experimentalist in optical activity at Cambridge University, considered the introduction of a new term in the France of that time to be a bold step,[23] since introducing new words into the French language was a jealously guarded prerogative of the Académie Francaise.

Pasteur, genius though he was, could not solve the problem of *precisely what* is required to make a molecule dissymmetric. The best he could do was to pose this question about the structure of the *d*- and *l*-tartaric acids:

"Are the atoms of the dextro-acid grouped on the spirals of a right-handed helix, or placed at the apices of an irregular tetrahedron, or arranged according to some one dissymmetric grouping or another? We cannot answer these questions. But what cannot be doubted is that the atoms are grouped in a dissymmetric order, which is not superposable on its image. It is no less certain that the atoms of the levo-acid show exactly the opposite dissymmetric grouping."[4]

The exact title of the two lectures from which this quotation is taken is "Recherches sur la Dissymétrie Moléculaire des Produits Organiques Naturels". Unfortunately, the word "dissymétrie", although always used correctly in the French literature, e.g., by Le Bel,[24] has been almost universally mistranslated as "asymmetry" in the American, English, and German literature.

The confusion between these two fundamentally different ideas is still so common that in 1974, in commemorations of the 100th anniversary of the van't Hoff–Le Bel hypothesis of the asymmetric carbon atom, a host of old errors were revived and a spate of new ones generated.

To clear up this confusion it may help to consider a couple of conspicuous examples from the literature. One fairly accessible source for a translation of Pasteur's lectures is Alembic Club Reprint No. 14, of the Chemical Society of London.[25] Beginning with Pasteur's title, the Alembic Club have everywhere mistranslated "dissymétrie" by "asymmetry".[26]

On May 24, 1912, Professor Alfred Werner delivered before the Société Chimique de France a lecture correctly reported as "Sur les Composés Métalliques à Dissymétrie Moléculaire".[27] The German abstractor gave it as "Über die Metallverbindungen mit molekularer Asymmetrie".[28] The American translator[29] not only got the title wrong, "Molecularly Asymmetric Metallic Compounds", but in at least a dozen places in the text used "asymmetric" for compounds which are clearly symmetric— some of them highly so.

From what has been said thus far, it is clear that nonsuperposability of object and mirror image is *not*, as sometimes alleged, a test for asymmetry. If a molecule, crystal, or other object has one or more proper axes of symmetry as its *only* symmetry element(s), the object and mirror image will not be superposable. The plagihedral quartz crystals mentioned previously, as well as the tris(ethylenediamine)cobalt(III) cations to be discussed in the next section, are enantiomorphous, even though highly symmetric. Nonsuperposability of object and mirror image is a test for *dissymmetry* (*chirality*), whether the object is symmetric or asymmetric.

B. van't Hoff's First and Second Cases of Optical Activity

1. The First Case

Success in showing how dissymmetry could be used to solve problems in isomerism was first achieved when van't Hoff[30-32] and Le Bel[24] independently suggested the hypothesis of the "asymmetric carbon atom" in 1874. Their achievement lies
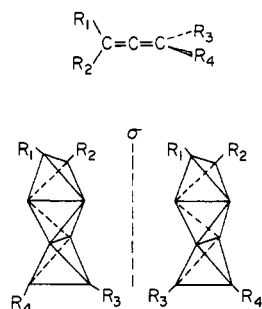
Figure 3. Asymmetric dissymmetric allene derivative.

Figure 4. Symmetric dissymmetric allene derivative.

Figure 5. Symmetric dissymmetric allene derivative, C_2 axis.

Figure 6. Symmetric dissymmetric spirane, C_2 axis. Two asymmetric carbon atoms.

primarily in the fact that they were able to relate this idea very specifically to the problems of isomerism and optical activity.

van't Hoff favored a tetrahedral arrangement of groups around a central carbon atom,[33] but Le Bel favored a configuration with the asymmetric carbon atom at the apex of a squarish pyramid.[36,37] The development followed here is that of van't Hoff. Professor Snelders has published a most interesting account of the rather cool reception afforded van't Hoff's attempts to rewrite planar chemical formulas in three dimensions.[38]

In his first pamphlet in Dutch in September 1874,[30] as well as in the two versions in French which he published almost immediately,[31,32] van't Hoff dealt only with the first case of optical activity, involving the asymmetric carbon atom.

van't Hoff's final definition of an asymmetric carbon atom is given in "La Chimie dans l'Espace", published in May 1875. A carbon atom surrounded by four different univalent (French, monatomique; German, einwertig) groups was defined by van't Hoff to be an "asymmetric carbon atom".[39,40] It is very clear that he made this definition depend on the absence of a plane of symmetry in tetrahedral compounds like $C(R_1R_2R_3R_4)$.[41,42]

In applying this definition originally to some 15 compounds, van't Hoff included such groups as OH, NH_2, and COOH under the term "univalent". Since the application of van't Hoff's definition requires that the groups have free rotation in space, and since the groups just mentioned have no right- or left-handed character (chirality), it is perhaps better to call them scalar groups.

As originally applied by him, this definition causes no difficulty. However, without further development, as will be seen later (section II.E.1), it is not generally satisfactory. Even today, as Hirschmann and Hanson point out, there is no generally accepted definition of an asymmetric carbon atom.[43]

Since there is confusion in the literature as to whether van't Hoff thought an asymmetric carbon atom could exist in a ring, and since this has had an effect on the search for improved nomenclature in organic stereochemistry, asymmetric carbon atoms in cyclic compounds will be discussed subsequently (section II.D).

2. The Second Case

In "La Chimie dans l'Espace" van't Hoff proposed his "second case of optical activity".[44] As he stated explicitly, this second case does not depend on the presence of an asymmetric carbon atom.[44,45] Unlike the first case, specific examples for the second were unknown. To illustrate it, van't Hoff therefore used a hypothetical allene derivative with tetrahedra connected along an edge representing double bonds (Figure 3).

In addition to this asymmetric dissymmetric allene, van't Hoff considered a symmetric dissymmetric one (Figure 4). That it is symmetric—i.e., that it possesses at least one true element of symmetry—can be seen by grasping the left end of the molecule (Figure 5) and rotating it 90° out from the page. It will then look as shown in the Newman projection, in which the twofold axis of symmetry, C_2, is seen. Errors on the symmetry of this kind of allene derivative and the related spiranes (Figure
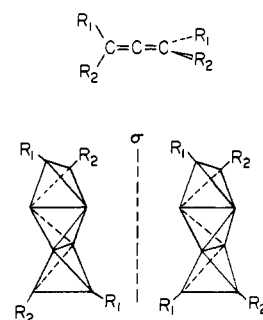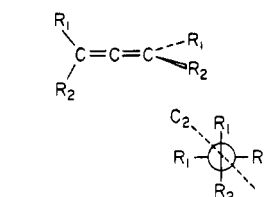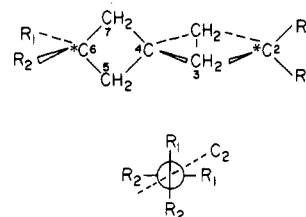
6) are so common that this exercise is well worth doing. That the symmetric chiral spirane of Figure 6 has two asymmetric carbon atoms will be discussed in the next part (section II.C).

The great Dutch physical chemist F. M. Jaeger, who several times corrected Pope's misunderstandings and errors about symmetry,[46,47] has the symmetry of the allene derivative of Figure 5 wrong in his masterly "Lectures on the Principle of Symmetry".[48] In his lectures at Cornell University[49] he refers only to the asymmetric allene of the type shown in Figure 3, but errs in asserting that a spirane of the type shown in Figure 6 is asymmetric. That it has a C_2 axis can be seen by doing the exercise recommended for the symmetric allene derivative.

That these are not simply errors of a distant past is shown by the fact that at least two contemporary textbooks of organic chemistry, both in their third editions, state incorrectly that allenes of the type of Figure 5 are asymmetric. Mislow has unusually detailed information on the symmetry of allene and its derivatives.[50]

Like the asymmetric allene derivative, the symmetric one has no asymmetric carbon atom, and again like the asymmetric one, its potential optical activity is due to the fact that it is dissymmetric (chiral).

Why mention the second case at all? Although the "asymmetric carbon atom" was a great idea[51] which stimulated an enormous amount of research, it had some bad effects:

(a) By about 1900–1910 it was being asserted that an asymmetric carbon atom was essential for optical activity. The situation was so bad that Ossian Aschan, a theoretician, appealed for the aid of synthetic organic chemists, since they are best fitted "to prove the erroneousness of the dogma that in each [optically] active body at least one asymmetric carbon atom must be present".[52]

(b) It focussed the chemists' attention on the wrong thing. Since van't Hoff had defined an asymmetric carbon atom as one having four different univalent (i.e., scalar) groups—in consequence of which a plane of symmetry could not be passed

TABLE I. van't Hoff's Classification of Dissymmetric (Chiral) Molecules

asymmetric molecule $\begin{cases}$ with one (or more) asymmetric C atom(s)   case I
or
without asymmetric C atom $\end{cases}$

symmetric molecule   with an axis (or axes) of symmetry only $\Big\}$ case II

C_3 and 3 C_2 oxes
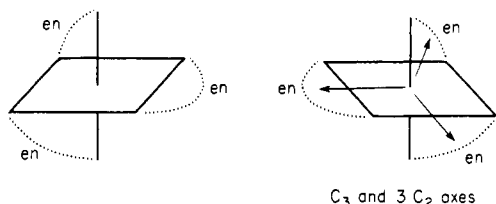
Figure 7. Cobalt(III) cation, point group D_3.

through it—chemists had begun to think that *dissimilarity* of four groups is essential for optical activity. Jaeger,[53] however, pointed out that by far the greatest optical activity—some 10–100 times that in molecules with an asymmetric carbon atom—is found in ions like the tris(ethylenediamine)cobalt(III) cation (Figure 7), which have all the substituents the same.

This cobalt(III) cation, taken from Werner's papers,[27,29] has the same high symmetry, point group D_3,[53] as the hemihedral quartz crystals referred to earlier: three C_2 axes lying in a plane, 120° apart, with a C_3 axis perpendicular to all three of them. In Figure 7 one C_2 axis is shown going to the left in the plane of the paper, one going above and behind the plane, and one going down and to the front of this plane. To avoid cluttering the figure the C_3 axis is omitted.

van't Hoff's two cases of optical activity show, then, that neither symmetry nor asymmetry should be invoked as the necessary condition for optical activity. Instead, the *necessary condition* is what Pasteur had termed *molecular dissymmetry*. van't Hoff correctly classified dissymmetric (chiral) molecules as shown in Table I.

As Pope said so well, *an* (i.e., *one*) asymmetric atom is a convenient molecular sign of molecular enantiomorphism, *not* the cause of optical activity.[23]

C. A Historic Controversy: (4-Methylcyclohexylidene)acetic Acid and Allene Derivatives. The Spiranes

Having considered Pasteur's criterion for optical activity and van't Hoff's second case of it, let us now consider a historic controversy which illustrates many points previously mentioned. This controversy, in which Pope was embroiled for about 15 years, has not yet, I believe, been satisfactorily resolved and remains as a trap in Lowry's monumental work.[54] The resolution of the difficulties illustrates that one must not rely too much on geometry in assessing equivalence of nuclei and that both (4-methylcyclohexylidene)acetic acid derivatives and the corresponding spiranes differ essentially from allenes in having one asymmetric carbon atom in the former case and two in the latter.

Early attempts to resolve allene derivatives into the optically active enantiomers, which van't Hoff said should exist, all failed.[54] In fact it was not until 1935, after Lowry's book had gone to press, that two groups, one at Cambridge University and one at Harvard, succeeded in resolving such derivatives.[55,56]

Perkin, Jr., Pope, and Wallach recognized that the essence of the dissymmetric allene arrangement—insofar as the spatial geometry is concerned—is to have three atoms in a row with terminal groups in two different planes, as we saw earlier in van't Hoff's representation (Figure 3). They therefore made and resolved (4-methylcyclohexylidene)acetic acid (Figure 8).[57,58]

However, their claim to have made the first optically active compound without an asymmetric carbon atom was disputed by Everest and Marsh. Everest relied on "the generally accepted theory that the optical activity of a substance is due to the
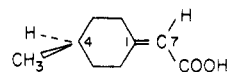

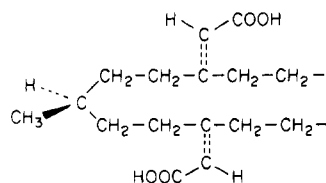**Figure 8.** (4-Methylcyclohexylidene)acetic acid.

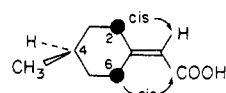**Figure 9.** Everest's configuration for (4-methylcyclohexylidene)acetic acid.

Figure 10. Chemical nonequivalence of C-2 and C-6 atoms in (4-methylcyclohexylidene)acetic acid.
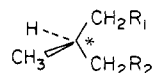
Figure 11. C-4 as asymmetric atom in (4-methylcyclohexylidene)acetic acid.

presence in it of an asymmetric atom", claiming, moreover, that C-4 is asymmetric.[59] To support this contention, Everest displayed the acid as shown in Figure 9, asserting that, since C-4 has four different groups attached, it is asymmetric.

Rejecting the reply by Perkin and Pope,[60] Marsh partly backed Everest's rejoinder,[61] agreeing that he "disposes effectively of the claim that a new type of asymmetry exists which is not dependent on the presence of an asymmetric atom in the molecule".[62,63] Marsh took issue not only with Perkin and Pope but also with Everest. Using the definition of the asymmetric carbon atom as one in which "exchange of place of two of the groups attached to it" produced an image not superposable on the original—a definition which he had proposed in 1888[69]—he said that there were three asymmetric carbon atoms in (4-methylcyclohexylidene)acetic acid, C-4, C-1, and C-7 (Figure 8).

The controversy lasted till the mid-twenties, with most of the participants—notably at the first Solvay conference[46]—favoring Sir William's interpretation. The principal value of the controversy was to bring out three things: First, there was *no* satisfactory definition for an asymmetric carbon atom. Second, it was necessary, in determining potential optical activity, to go back to Pasteur's idea and consider the symmetry of the molecule as a whole. Jaeger, in fact, castigating organic chemists for using the plane of symmetry as a test for possible optical activity, proposed the motto: "Let us return to Pasteur".[70] Third, as Jaeger repeatedly pointed out, the so-called asymmetric carbon atom is such a *special* case, so full of *traps* for the unwary, that it is of very doubtful pedagogical value.[71] Unfortunately, Jaeger's advice was not commonly followed for over 40 years.

Finally, however, with respect to the presence of an asymmetric carbon atom in (4-methylcyclohexylidene)acetic acid, it seems to me that Everest was right and Sir William was wrong. Considering the compound again (Figure 10) we see that C-2, being cis to H, will be chemically different from C-6, which is cis to COOH. Since one would expect differences in the proton magnetic resonance spectra of the C-2 and C-6 methylene groups, we can represent the groups to which C-4 is attached

Figure 12. Oximes, hydrazones, and semicarbazones: 4-carboxy-cyclohexylidene derivatives.

Figure 13. Generalized 4-X-4-Y-cyclohexylidene derivatives with C-4 an asymmetric carbon atom.

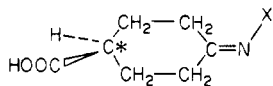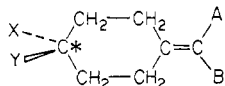as in Figure 11, so that C-4 is, indeed, an asymmetric carbon atom.

The chemical nonequivalence of C-2 and C-6 can also be seen from the fact that these nuclei are diastereotopic—an approach that will be used shortly in discussing the spiranes.[73]

Besides (4-methylcyclohexylidene)acetic acid, Lowry[54] lists some oximes, hydrazones, and semicarbazones derived from a common ketone. They can all be represented as shown in Figure 12. All contain a cyclohexylidene ring, and concerning all of them, as well as the o-carboxyphenylhydrazone of methyltrimethylene dithiocarbonate, Lowry says that they "do not contain an asymmetric carbon atom in the sense of van't Hoff's definition".[54]

However, I believe that the conclusion that C-4 is a chiral center in cyclohexylidene derivatives is general, and that, in fact, in the light of modern knowledge with respect to chemical equivalence, the C-4 atom in compounds of the types shown in Figures 12 and 13 should always be, in principle, asymmetric. These considerations cast doubt on the equivalence of cyclohexylidene and allene derivatives as proposed by Perkin, Jr., Pope, and Wallach.[57,58]

Because of the geometrical similarity of spiranes of the type shown in Figure 6, in which two rings replace two double bonds, to the allenes of Figure 5, such spiro compounds have been considered to be examples of symmetric dissymmetric compounds without asymmetric carbon atoms. Many of these spiro compounds have been synthesized and resolved into optically active forms.[54] However, consideration of the spiro[3.3]heptane displayed in Figure 6 shows that C-2 and C-6 are asymmetric.

Since the C_2 operation interchanges the C-1 and C-7 methylene groups, and simultaneously the C-3 and C-5 methylene groups, the C-1 group is equivalent to the C-7 and the C-3 to the C-5. (Within a given methylene group, however, the protons are anisochronous[74] because of their physical (spatial) relations to R$_1$ and R$_2$.)

The C-5 methylene and C-7 methylene groups are diastereotopic; i.e., they cannot be interchanged by any symmetry operation.[77] They are in two different physical environments and therefore are, in principle, chemically nonequivalent. Thus the C-6 atom is attached to four different groups and so is asymmetric. Because the C-1 and C-3 methylene groups are likewise diastereotopic, C-2 is also asymmetric.

On the basis of a proposed definition of an asymmetric carbon atom quite similar to that given by Marsh,[69] it has been suggested that C-4 of the spiro[3.3]heptane of Figure 6 is asymmetric. However, since C-1 is equivalent to C-7, and C-3 to C-5, this does not seem correct. This agrees with Hirschmann and Hanson's judgment that the proposed definition "does not seem to be a suitable basis for a general definition of a chiral center".[43]

Thus, although (disregarding conformational isomerism) a double bond may be *geometrically* equivalent to a ring, *physically*, 4-methylcyclohexylidene and the corresponding spirane derivatives differ essentially from symmetric dissymmetric allene derivatives in that the cyclohexylidenes have one asymmetric carbon atom and the spiranes have two.

The long axis through the C=C=C bonds of allene derivatives (Figures 3 and 4) and the long axis going through C-7, C-1,

and C-4 (in its average conformational position) of (4-methyl-cyclohexylidene)acetic acid are examples of a *chiral axis*.[78]

D. Asymmetric Carbon Atoms in Cyclic Compounds

J. E. Marsh, translator of the second edition of "La Chimie",[63–65] in which cyclic compounds are not treated, erred regarding van't Hoff's thinking concerning the possibility of an asymmetric carbon atom in a ring. On page 115 Marsh says:[65] "In the first place, with regard to asymmetry, of course no carbon situated in a closed chain can be combined with four different groups, but if it does not possess a plane of symmetry it will still be asymmetric." This quotation concerns the primacy of four different groups or the plane of symmetry, *when both are present*, in determining whether or not a carbon atom is asymmetric. This primacy is discussed in section II.E.1.a.

Comparison of Marsh's translation[65] with the original[64] shows that pages beyond 112 were added by Marsh. Moreover, from examining "La Chimie", its translation into German, and all the subsequent editions of "La Chimie" in English, French, and German, I believe van't Hoff never said this.

More important, both in his very first publication in Dutch[30] and in the French translation of it,[31] van't Hoff gives camphor, borneol, camphoric acid, terpinolene, and menthol—with figures of four of them having Kekulé's planar benzene ring—as examples of ring compounds containing asymmetric carbon atoms.

The isomerism of the hexane derivatives $C_6H_6(COOH)_6$, hydromellitic and isohydromellitic acids, is discussed briefly in his second principal work on stereochemistry, "La Chimie".[79] Because the differences between the configuration of benzene as a planar ring (Kekulé) and a prism (Ladenburg) had still not been resolved,[79] and because these acids, in 1875, seemed rather isolated cases, extensive discussion of cyclic compounds with asymmetric carbon atoms was postponed until the appearance of "Stéréochimie", which Meyerhoffer edited with van't Hoff's assistance.[67]

Perhaps van't Hoff's omissions of material in different editions, mentioned previously (section II.C),[63] misled Marsh. In what follows (section II.E.1.c), van't Hoff's explanation for optical inactivity in a ring having two asymmetric carbon atoms is given.

E. The Necessary Condition for Optical Activity

1. Historical: The Search for the Root of Dissymmetry: Asymmetric Atoms, Pseudoasymmetric Atoms, Diastereomeric Atom

Since errors about the criterion for optical activity, long ago laid to rest, are now recurring, we look at some of the major difficulties in the past. Historically there were two. First, although the van't Hoff–Le Bel concept of the asymmetric carbon atom was initially received with coldness,[80] in the enthusiasm eventually generated, Pasteur's criterion of molecular dissymmetry was overlooked.

Second, the effort to determine which symmetry elements could be present in a dissymmetric molecule met with several difficulties. All of the molecules in Figure 14 caused trouble.

a. "Pseudoasymmetric Atom". Diastereomeric Atom. This is illustrated in Figure 14A by a compound having a central carbon atom with four different groups attached, but possessing a plane of symmetry. In the notation used in Figure 14, h and i are scalar groups as defined previously (section II.B). The meaning of g$^+$ and g$^-$, which are groups with the same constitutional formulas but different stereochemical configurations, can be seen from the example par excellence 2,3,4-trihydroxyglutaric acid (THGA) (Figure 15).

This molecule was troublesome to van't Hoff, Fischer, Landolt, Mohr, and Pope, and has been a source of contention ever since. Hirschmann and Hanson note that contemporary authors dis-
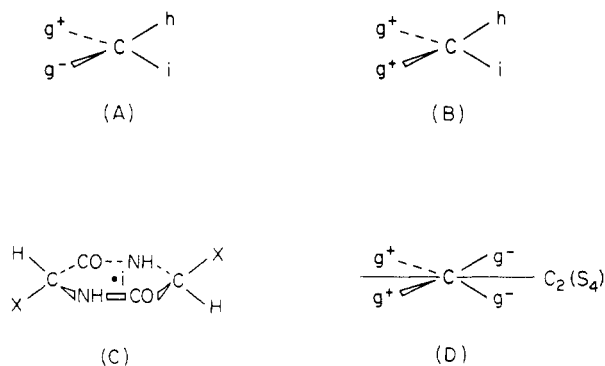
g^+ ‒ ‒ C ⟨ h / i

(A)

g^+ ‒ ‒ C ⟨ h / i

(B)

H \ / CO‒‒NH \ / X
>C< •| >C<
X / ⟍NH⟆CO / \ H

(C)

g^+ ‒ ‒ C ⟨ g^- / g^- ‒‒ $C_2 (S_4)$
g^+ / g^-

(D)

Figure 14. Which symmetry elements can be present in a dissymmetric molecule?

σ
H | OH
HOOC \ / C \ / COOH
C C
H / OH H / OH

Trihydroxyglutaric Acid

Configuration Left End

HOOC \ / COOH
)‒‒‒‒‒>C‒‒‒
H / OH

△
H OH

Configuration Right End

△ COOH
HO H

COOH
‒‒C<‒‒‒‒()
H OH
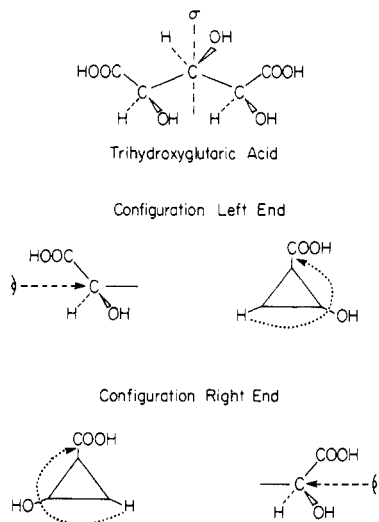
Figure 15. 2,3,4-Trihydroxyglutaric acid. Configuration of ends in optically inactive isomers.

σ^{xz}

◁ ‒ ‒ → H ‒ ‒ / C ⟨ ‒‒‒H ← ‒ ‒ ▷
CH₃ ⟍ C₂H₅ | H₅C₂ ⟍ CH₃

Figure 16. Reflection reverses handedness (chirality).

agree as to the proper designation of the central carbon atom in Figures 14A and 14B and give a resumé of differing views in the past.[43]

Basically there are two different problems. The first is the proper designation for a central carbon atom which has a plane of symmetry, even though attached to four different groups. The second problem is the number of isomers in a compound with n asymmetric carbon atoms. If the compound contains no element of symmetry, i.e., is asymmetric, it is well known that the number of isomers is 2^n.

van't Hoff thought[83,84] that molecules of the type C-$(R_1R_2R_3)C(R_4R_5)C(R_1R_2R_3)$, as in Figure 14A, could have only three isomers. The original text shows clearly that he believed the plane of symmetry when the molecule has the configuration of Figure 14A reduces it to the type of $C(R_1R_2R_3)C(R_4R_4)C$-$(R_1R_2R_3)$, which has three isomers.

Fischer showed van't Hoff[85] that there are four isomers, but erred himself as to the optically inactive pair. Unlike Le Bel, who 17 years earlier saw that $C(R_1R_2R_3)C(R_1R_2R_3)$ is inactive if the configurations of the ends are opposite,[24] and who also anticipated van't Hoff on this point, Fischer thought THGA is symmetric when they are the same. He overlooked the fact that reflection not only moves the groups but reverses their handedness (Figure 16).

Figure 15 shows the relations correctly. If, in Figure 15, the counterclockwise order of the H, OH, and COOH groups on the left end of the THGA is called g^-, then the clockwise order of these groups on the right end would be designated g^+.

Clearly, if the central atom of THGA were asymmetric, there should be 2^3 isomers. But, as Fischer says,[85] the middle carbon atom, in the inactive forms, "has lost its asymmetry". Hermann,
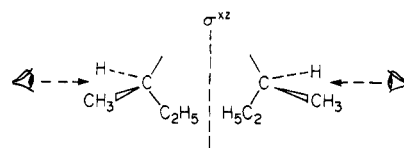
too, said that the symmetry of a compound like THGA depends only on the configuration of the end groups,[86] which lowers the number of isomers to 2^2.

Perhaps because van't Hoff's reasoning in arriving at the definition of an asymmetric carbon atom had been forgotten or misunderstood, the central carbon atom was sometimes called asymmetric in the 1890's. However, Landolt shows[87] that one must modify the formulas for the total number of isomers, number of optically active isomers, and number of optically inactive isomers if one says the central carbon atom is asymmetric "which in reality it is not (pseudo-asymmetry)".[88] The original sentence and its translation are given in ref 88.

Landolt returns to this again,[89] but then, referring to the configuration of Figure 15, he says, "In reality, however, this latter [the most central carbon atom] cannot be considered asymmetric, since, as a glance at the following configurational formulas shows, a symmetry plane can be passed through it and between the unlike radicals attached to it." The German text is given in the note, together with the sentences immediately preceding and following.

Like van't Hoff,[41,42,90] Landolt makes the existence of a symmetry plane through the central carbon atom of Figure 15 decisive. Moreover, he does not appear to be inventing a new name and designating this central carbon atom "pseudoasymmetric", as usually thought, but was calling the asymmetry attributed by some to the midmost carbon atom a "false-", "fake-", or "deceptively seeming-asymmetry". Apparently this is also Werner's view.[91]

Generally it has been overlooked that 3 years before he died van't Hoff himself resolved the controversy over the proper designation of the central carbon atom of the inactive form of THGA. In the third edition of "Die Lagerung", discussing the optically active and inactive THGA's, van't Hoff says[92] that Mohr[93,94] raised the objection that a precise examination of the central carbon atom shows it is asymmetric, since it is connected to four different groups. Cutting through Mohr's faulty argument, van't Hoff makes nonsuperposability of object and mirror image—Pasteur's criterion—decisive. Applying this to the molecule in Figure 15, he says that, since molecule and mirror image are superposable, there is no asymmetry.

The IUPAC 1974 recommendations[95] call the central carbon atom of a molecule C(hig⁺g⁻) a "pseudoasymmetric atom". For several reasons, the first being that, as we have just seen, the atom in C(hig⁺g⁻) is symmetric, I believe "diastereomeric atom" more appropriate.

The committee note (1): "The molecular structure around a pseudoasymmetric atom gives on reflection an identical (superimposable) structure." However, exactly the same is true with the symmetric molecule C(hik₂), and for exactly the same reason: both it and C(hig⁺g⁻) have a plane of symmetry. In fact, since this is the only symmetry element of either of them, they both belong to point group C_s.

The committee note also (2): "Compounds differing at a pseudoasymmetric atom belong to the larger class of diastereoisomers." (A diastereomer, or diastereoisomer, is a stereoisomer which is not the mirror image of the original;[96] or, as rule E-4.6 says, "Stereoisomers that are not enantiomeric are termed diastereoisomers."[95]) This reason for specially distinguishing the atom is cogent.

However, an etymologically unfortunate term, with unclear connotations, could be avoided were such an atom called a

"diastereomeric atom". This is etymologically correct, denotes exactly the effect of such an atom, and distinguishes it from C(hik$_2$). Although the central atoms in both C(hig$^+$g$^-$) and C(hik$_2$) are symmetric, and both molecules belong to the same symmetry point group, the latter has only one possible configuration.

How the NMR spectra of groups adjacent to a diastereomeric atom are affected is explained by Mislow and Raban,[97] whose analyses are confirmed by the results of Prelog and co-workers.[51,98]

b. Asymmetric Atom. Although one often thinks of an asymmetric atom as one of the type C(hijk), Figure 14B shows an asymmetric molecule with two identical groups attached to a central carbon atom.

A molecule like this could be one member of the racemic pair of the THGA just discussed. Since two of the four groups are the same, it looks at first as though it should have a plane of symmetry, as in C(hik$_2$). Reflection of C(hig$^+$g$^+$), however, gives C(hig$^-$g$^-$), not superposable on the original.

Apparently because of difficulties in nomenclature which have occurred when chiral groups are attached to a central carbon atom, the IUPAC committee[95] have decided to return to van't Hoff's original definition of an asymmetric carbon atom, in which the groups around a central carbon atom are scalar (section II.B). Although there is some advantage in doing this, this definition has the disadvantage that no name is suggested for the central carbon atom of Figure 14B. Yet, from the point of view of symmetry, as well as physically in its effect on polarized light, the midmost carbon atom there is just as asymmetric as the one in C(hijk).

c. Molecules with Only a Center of Symmetry, i. For many years chemists could not understand why the *trans*-diketopiperazines (Figure 14C), which have two asymmetric carbon atoms but no plane of symmetry, are optically inactive. Complicated explanations were invented.

Ladenburg,[99] noting that such a molecule can be superposed on its mirror image, even though it has no mirror plane, invented the symmetry element "pseudosymmetry plane". For this he was rather sternly rebuked by the eminent crystallographer Groth,[100] founder and editor of the *Zeitschrift für Krystallographie und Mineralogie.*

Explaining that Ladenburg's method would only lead to confusion with respect to the rotation–reflection axes already well-known to crystallographers and mathematicians, Groth said that the reason for superposability of object and mirror image was the presence of the sole symmetry element, the center of symmetry, *i.* Since chemists were obviously floundering, Groth recommended that the elements of symmetry be taught as a regular part of the chemistry curriculum—a suggestion not implemented for about 75 years.

Interestingly, van't Hoff, in discussing the optical activity of these same *trans*-diketopiperazines many years later,[101] does not advert to the presence of a center of symmetry but uses the equivalent combined operation, σC_2. This works because $\sigma C_2 = C_2\sigma = S_2 = i$.[8a,102] van't Hoff explains that the optical inactivity of molecules similar to that of Figure 14C is general, because the two asymmetric carbon atoms in the ring have enantiomorphous configurations relative to one another.

d. Molecules with Only an S$_4$ Axis. Asymmetric Atoms—Again. Molecules of the kind of Figure 14D really confused chemists, because as just pointed out, they were unfamiliar with improper axes of rotation. Two of those confused were Aschan[52]—the one who asked synthetic organic chemists for help—and Mohr.[93]

Mohr corrected Fischer's errors with respect to THGA,[85] but made a few himself. For example, he begins his article by saying, "According to Pasteur, the optical activity of organic compounds is a consequence of the asymmetry of the molecule"—a statement Pasteur nowhere makes.
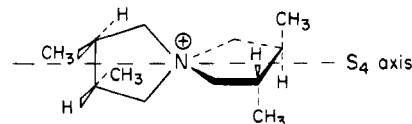

Figure 17. Tetramethylspirobipyrrolidinium ion with S_4 axis.

Mohr, whose article is entitled "On the Theory of the Asymmetric Carbon Atom", believed that the molecule of Figure 14D has a C_2 axis as its only symmetry element. Since such molecules, like asymmetric ones, are not superposable on their mirror images (section II.A) Mohr thought the two classes could be subsumed by defining an asymmetric carbon atom as one through which you could not pass a plane of symmetry. However, in private correspondence, Aschan pointed out to Mohr that this molecule *is* superposable on its mirror image, which led Mohr to exclaim: "The fact that an asymmetric model can be brought into congruence with its mirror image seems immediately to be contradictory and unthinkable."

The difficulties which he encountered led Mohr to suggest, in the last paragraph of his article, that since his own more extensive development of the Le Bel–van't Hoff principle actually brought one closer to Pasteur's idea of nonsuperposability of object and mirror image as the root of optical activity, that might be the better way to go. In this he anticipated by 20 years Jaeger's motto, "Let us return to Pasteur",[70-72] advice warmly seconded by Delépine in his address before the Société Chimique de France, celebrating the fiftieth anniversary of the theory of the asymmetric carbon atom.[103]

The reason, unknown to Mohr and Aschan, that the molecule of Figure 14D is superposable on its mirror image, and therefore optically inactive, is that there is an S_4 axis coincident with the C_2. Because this axis is less familiar and somewhat troublesome, it is considered briefly in the appendix (section V).

By 1920 a small percentage of chemists recognized that a molecule with an S_4 axis should be optically inactive. It was not, however, until 1955 that a molecule with an S_4 axis as its *only* symmetry element of the second kind was synthesized—the tetramethylspirobipyrrolidinium ion of Figure 17.[104] As predicted, it is optically inactive.

We can summarize the four major difficulties encountered in the search for the root of dissymmetry by saying that from the time of the van't Hoff–Le Bel hypothesis in 1874—in spite of the fact that mathematicians and crystallographers knew the root of dissymmetry by at least 1890—it took almost 50 years for a small percentage of chemists to recognize that, in terms of the symmetry elements of the molecule, Pasteur's criterion for optical activity required the *absence* in the molecule of a plane of symmetry, σ, a center of symmetry, i, and an improper axis of rotation, S_n.[105] This, the necessary condition for optical activity, is summarized in Table II. Unfortunately by 1925 both literature and textbooks were so full of errors on the subject that both are still suffering.

2. Contemporary

A basic confusion in many contemporary textbooks arises from a failure to differentiate *clearly* two entirely different questions: 1. Are there elements of symmetry present? 2. Is the molecule superposable on its mirror image? If the answer to the first question is no, the molecule is asymmetric; if yes, symmetric. If the answer to the second question is no, the molecule is dissymmetric (chiral), but, as we have seen previously, in the case of quartz crystals and complex cobalt cations, may be highly symmetric. If the answer is yes, the molecule is nondissymmetric (achiral). This matter is further treated in Table III, which classifies molecular point groups according to these two questions.

Applying this classification to some common molecules, we see that the asymmetric allene of Figure 3 belongs to point group

TABLE II.   Necessary Condition for Optical Activity (Small Molecules in the Isotropic Liquid Phase)

nonsuperposability of object
  and mirror image[a] (Pasteur's
  Dissymétrie Moléculaire)—requires
  the *absence* in the molecule of:

| a plane of symmetry, σ ($=S_1$)
a center of symmetry, i ($=S_2$)
an improper axis of rotation in the strict sense, S_n | an improper axis of rotation, S, in the broad sense |

[a] Nonsuperposability of object and mirror image requires the presence of right- and left-handed forms. Since the mirror operation turns a right-handed form into a left-handed one, and vice versa (see Figure 16), an object with a mirror plane as an element of symmetry cannot exist simultaneously in right- and left-handed forms. Two of the improper axes, σ and S_n, involve reflection. For S_4 see Appendix V.D. How the third improper axis, i, involves σ is explained in section II.E.1.c, especially ref 102.

C_1; it is asymmetric dissymmetric. The symmetric allene of Figure 4 and the spirane of Figure 6 belong to point group C_2; they are symmetric dissymmetric.[7,8a,106] The hemihedral quartz crystals of Figure 2 and the cobalt(III) cation of Figure 7, belonging to point group D_3, are also symmetric dissymmetric.

(+)-Tartaric acid and (−)-tartaric acid, often referred to in relatively recent literature as asymmetric because they lack a plane of symmetry, belong to point group C_2. *meso*-Tartaric acid, having a plane of symmetry as its only symmetry element, belonging to point group C_s, and therefore of lower symmetry than objects in point group D_3, is symmetric nondissymmetric. (In all of the above examples one can read "chiral" for "dissymmetric" and "achiral" for "nondissymmetric".)

Like *meso*-tartaric acid, the two inactive forms of THGA (one shown in Figure 15) have only a plane of symmetry (point group C_s). Lacking any element of symmetry, the racemic pair belong to C_1.

2,4-Dihydroxyglutaric acid is an example of a C(R$_1$R$_2$R$_3$)C-(R$_4$R$_4$)C(R$_1$R$_2$R$_3$) molecule mentioned earlier (section II.E.1.a). It has the same number of isomers as tartaric acid. Like the meso forms of tartaric acid and THGA, the inactive isomer is in point group C_s. Unlike the racemic pair of THGA, however, the racemic pair of this molecule resembles the racemic pair of tartaric acid in having a twofold axis of symmetry and belonging to C_2.

Finally, as stated in the first discussion of asymmetric atoms in this section, the molecule of Figure 14B is asymmetric. The reader might like to try assigning the following symmetric dis-
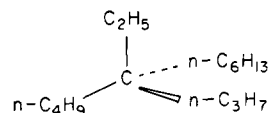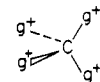
$$
\begin{array}{c}
C_2H_5 \\
|\\
\cdots n-C_6H_{13}\\
n-C_4H_9 \diagup C \diagdown n-C_3H_7
\end{array}
$$

Figure 18. *n*-Butylethyl-*n*-hexyl-*n*-propylmethane, asymmetric and optically inactive.

symmetric molecule, assumed to be tetrahedral, to its point group(s?).

$$
\begin{array}{c}
g^+ \diagup g^+ \\
g^+ \diagup C \diagdown g^+
\end{array}
$$

F. Potential Optical Activity

The term "potential optical activity" has been used two or three times. What this means can be seen from this carefully engineered asymmetric molecule, *n*-butylethyl-*n*-hexyl-*n*-propylmethane (5-ethyl-5-propylundecane) (Figure 18). Between 280 and 580 nm, neither enantiomer shows any detectable rotation of the plane of linearly polarized light.[107,108]

This molecule is important for two reasons. First, it shows that dissymmetry of molecular configuration is not a sufficient condition for optical activity, even when the molecule is also asymmetric. In this particular case it is clear that the root of the difficulty is that the dissymmetry of *electronic* configuration is very low; i.e., the electronic interactions of the four alkyl groups with linearly polarized light are sufficiently similar that no rotation is observable. Moreover, by simply lengthening each alkyl chain, one could bring the specific optical activity below any previously prescribed level.

Jaeger was among the first to point out that dissymmetry of atomic configuration is a secondary factor in optical activity, and that dissymmetry of electronic resonators is primary.[109] Although we have been following a phenomenological explanation of optical activity as simply a rotation of the plane of linearly polarized light, and shall continue to do so, modern views of optical activity are based on the absorption of energy, usually by the electrons of a molecule.[18,20,21,110,111] Foss discusses very well the relation between electronic absorption and the ordinary index of refraction, as well as that between circular dichroism and optical activity.[112]

The second reason for the importance of the molecule of Figure 18 is that it points up a rather serious imprecision in IUPAC rules E-4.1 note (2) and E-4.2 note (1), especially the former. This says, "All chiral molecules are molecules of optically active compounds . . . There is a 1:1 correspondence

TABLE III.   Classification of Objects by Symmetry

I, true symmetry elements present?	designation	point groups	II, reflective symmetry present? (i.e., object superposable on mirror image?)	designation
none	asymmetric	C_1		
		C_n		chiral (dissymmetric)
only $C_n(s)$ ($n \geq 2$)		D_n T O J	no	(capable of existing as pairs of enantiomers)
	symmetric			
only σ		C_s		
only i		C_i		achiral (nondissymmetric)
S_4, highest element of symmetry[a]		S_4	yes	(objects constitute the homomers)
various combinations of i, C's, σ's, and S's		all others		

[a] σ and i can be considered S_1 and S_2, respectively. If n is odd, the presence of S_n *requires* the presence of both C_n and σ_h, and objects with these elements of symmetry belong to point groups C_{nh} and D_{nh}. S_4 is the lowest improper axis of rotation that can exist without the presence of either σ or i. Axes and point groups of order higher than S_4 are possible.
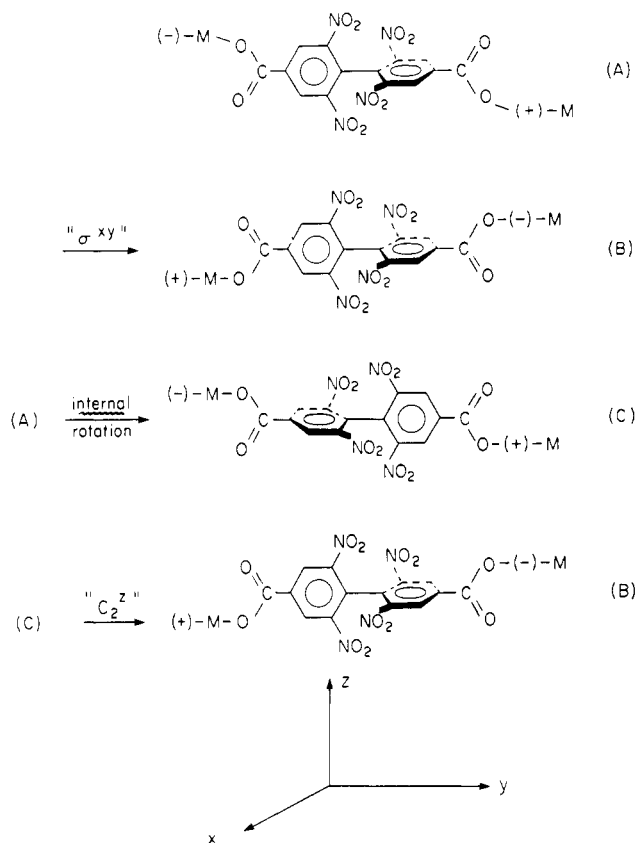
Figure 19. (+)-Menthyl (−)-menthyl 2,6,2′,6′-tetranitro-4,4′-diphenate, asymmetric and optically inactive.

between chirality and optical activity."

In accordance with the usual rules for correct use of English, to assert a 1:1 correspondence between chirality and optical activity is to say that chirality is not only the necessary but also the sufficient condition for optical activity, which is simply untrue. There is a 1:1 correspondence between chirality and enantiomorphism. That there is not a 1:1 correspondence with enantiomorphism and optical activity is explicitly stated in the quote from Hobden at the beginning of section III. The rules objected to could be corrected by saying that chiral compounds may be, or are potentially, optically active.

G. The Sufficient Condition for Optical Activity

Until 1954 it was thought that the necessary condition for optical activity was also sufficient, aside from accidental inactivity or racemic mixtures. However, Mislow suggested the possibility of optical inactivity in an asymmetric molecule—an inactivity having a cause different from that of Wynberg's molecule.[113] In 1955 Mislow and Bolstad synthesized one in a configurationally pure form (Figure 19A) and found it to be optically inactive.[114]

Although Mislow and Bolstad correctly explained the optical inactivity of this molecule, so many rather esoteric incorrect explanations have since been advanced that it is worth considering the reason again. Instead of inventing nonexistent symmetry elements or operations, it is better simply to admit the molecule is asymmetric. Why is it inactive?

Following the convention for the X, Y, and Z axes (Figure 19), reflect the molecule in the XY, or floor, plane. The chiral menthyl groups, M, reverse their handedness (chirality), and the result is the nonsuperposable mirror image of the molecule (Figure 19B).

Now start over, but this time, keeping the menthyl groups fixed as in (A), do an *internal*, 90° *clockwise* rotation of both phenyl rings. The new representation (C) is actually the same as the nonsuperposable mirror image (B). To see this, turn the molecule

in this new representation (C) end for end about the Z (vertical) axis.

The result is that an *internal rotation* by the phenyl groups acting in concert, which can easily occur, gives a conformation (C = B) which is the nonsuperposable mirror image (B) of the original molecule. The same result will occur if the phenyl rings rotate concertedly in the opposite direction.

Therefore, in any aggregate of molecules large enough to be examined for optical activity in a polarimeter, there exists one set of torsional enantiomers rotating the plane of linearly polarized light in one direction, because they are asymmetric. However, there exists a second set, of operationally equal size, which are mirror images of the first, rotating the plane of linearly polarized light in the opposite direction. This causes the observable activity to go to zero, just as though we had a 50:50 mixture of a permanent racemic mixture, such as racemic tartaric acid.

The necessary condition for optical activity is given in Table II. Since Mislow and Bolstad came up with their remarkable result in 1955, explicit statements of the sufficient condition for optical activity have become very hard to find.

If the material is in thermodynamic equilibrium (i.e., mechanical, thermal, and chemical equilibrium), and if it consists of only one chemical species, the necessary condition is also the sufficient, provided, first, that the material is not optically inactive by accident, and, second, that the material is not composed of transient enantiomeric pairs which easily interconvert so that their measured optical activity averages to zero by external compensation.

Wynberg's molecule (Figure 18) is an example of "accidental" inactivity; Mislow and Bolstad's molecule (Figure 19) is an example of inactivity due to transient enantiomeric pairs.[115] Ordinary racemic mixtures are excluded from consideration because permanent enantiomeric pairs constitute two chemical species.

H. Homomer

Although there exist names for an object or molecule which *cannot* be superposed on its mirror image, namely, enantiomorph or enantiomer,[116] there seems to be no noun to describe an object which *can* be superposed on its mirror image. To avoid the circumlocutions necessary to describe such objects or molecules, I propose the noun "homomer". Homomorph or nonenantiomorph would be possible, but the former has been preempted by biologists, the latter by crystallographers, with rather different meanings.

Section II of this paper has emphasized the importance of enantiomorphism of configuration (molecular dissymmetry—chirality) for possible optical activity in both symmetric and asymmetric molecules. That enantiomorphism of configuration is not the necessary condition for optical activity in crystals will be shown in the next section and for nematic liquid crystals in section IV.

III. On the Possibility of Optical Activity in Nonenantiomorphous Crystals: J. Willard Gibbs

"It is hoped that this observation will finally eradicate the notion that optical activity is exclusively related to enantiomorphism."

M. V. Hobden[118]

Although awkward, the term nonenantiomorphous crystal is standard in the crystallographic literature, meaning a crystal which can be superposed on its mirror image.[119] As mentioned in the preceding section, Pasteur erroneously concluded that enantiomorphism is the necessary condition for optical activity in crystals. Since the conditions for optical activity in crystals were not elucidated until much later, it is not surprising to see

van't Hoff saying that neither optically active molecules nor crystals possess a plane of symmetry.[120] Unfortunately, this error is repeated in Lowry's work[121] and is found in some contemporary texts. Lowry's error arose from a lack of familiarity with the work of Gibbs, Pockels, Szivessy, and Born.

It was shown in section II that for small molecules in the isotropic liquid phase (neat liquid or solution) the necessary condition for optical activity is molecular enantiomorphism (chirality). For crystals the situation is fundamentally different, and Jerephagnon and Chemla[122] distinguish three factors: (1) molecular enantiomorphism: rotation of the plane of linearly polarized light as exhibited by small molecules in the isotropic liquid phase (section II); (2) structural rotatory power: rotation of the plane of linearly polarized light as exhibited, e.g., by sodium chlorate crystals, which are optically inactive in solution (section III.E); (3) a polar longitudinal effect (appendix V.C). The latter two effects occur in regular three-dimensional arrays.

A. Gibbs's Predictions (1882)

The first to show that *enantiomorphism* is *not* essential for optical activity in crystals was J. Willard Gibbs.[124] In 1882 he wrote a paper in which, applying Maxwell's electromagnetic theory, he proposed that crystals belonging to two nonenantiomorphous classes of the tetragonal system, $S_4(\bar{4})$ and D_{2d}-$(\bar{4}2m)$, *could* be optically active.[125] Crystals of these classes are called (optically) uniaxial because they have only one optic axis.

Those not familiar with the Schoenflies notation[125] can tell from the International crystallographic notation—in which *m* stands for a mirror plane of symmetry—that crystals belonging to D_{2d}, as well as to two more classes to be mentioned in the next part of this section, have planes of symmetry, and can therefore be superposed on their mirror images. They are homomers (section II.H). Objects belonging to point group $S_4(\bar{4})$, although having an S_4 axis and a C_2 axis necessarily coincident with it[129] as their only true symmetry elements, can also be superposed on their mirror image, as shown in appendix V.D.

It was noted (section II.A) that Biot's discovery of dextro- and levorotatory α-quartz was made using sections cut *perpendicular* to the optic axis. Since optical activity is *forbidden by symmetry* along the optic axis in crystals belonging to classes $D_{2d}(\bar{4}2m)$ and $S_4(\bar{4})$, Gibbs's predictions mean that, in crystals in these two uniaxial classes, the structural rotatory power shows itself along crystal axes perpendicular to the optic axis.

Some time ago Fedorov proposed that, by a still further extension of the concept, optical activity might be observed in classes $C_{3v}(3m)$, $C_{4v}(4mm)$, and $C_{6v}(6mm)$.[131-133] Since the purpose of this section of this paper is primarily to call attention to the possibility of optical activity in nonenantiomorphous crystals, Fedorov's suggestion is discussed briefly later (Appendix V.C).

B. Pockels' Predictions (1906)

In 1906 Pockels, agreeing with Gibbs's analysis, pointed out that, besides two of the classes of the tetragonal system, there should be two more nonenantiomorphous classes in which crystals might be optically active, $C_s(m)$, of the monoclinic system, and $C_{2v}(mm2)$, of the orthorhombic system.[135] Crystals of these classes are called biaxial, since they have two optic axes.

The contemporary approach to determining the possibility of optical activity in an ordered, three-dimensional array is to apply tensor theory, obtaining some form of *optical gyration tensor*[122,136-138] or the related *scalar parameter* of *gyration* (Appendix V.C.2).[136,140] When the symmetry operations of each of the 32 crystal classes are applied in turn to the optical gyration tensor, the results show whether or not optical activity is possible in a given crystal class.

This has been done by Szivessy,[139] Born,[140] Mathieu,[141] Landau and Lifschitz,[142] and Ramachandran and Ramaseshan,[143] all of whom agree with Gibbs and Pockels. That optical activity is possible in the four classes of nonenantiomorphous crystals mentioned is referred to in contemporary textbooks of crystallography and crystal physics.[119,144,145]

The phenomenological approach adopted in section III.E-G, however, does not require one to go through this tensor analysis to understand the verification of Gibbs's and Pockels' predictions.

C. Sommerfeldt's Erroneous Report (1906)

In 1906 Sommerfeldt erroneously reported that crystals of the β form of mesityl oxide methyl oxalate dimer (obviously a
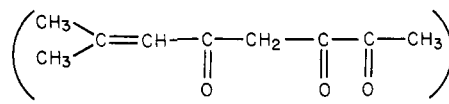
misnomer in modern terminology) are an example of optically active crystals of point group $C_s(m)$.[146] Although disputed by Voigt,[147] this error has gotten into the literature, as well as into a number of texts, via Szivessy.[148] This is discussed further in Appendix V.B.

D. Verification of Gibbs's Predictions: Hobden, 1967 and 1968

Futama and Pepinsky, in a note which seems to have been often overlooked,[149] were apparently the first to report optical activity in a nonenantiomorphous crystal, a ferroelectric of class $C_s(m)$—a biaxial crystal. However, since the problems of observing optical activity in nonenantiomorphous crystals can be more readily understood by considering uniaxial crystals, this group will be discussed first. The discovery by Futama and Pepinsky will be treated later (section III.G).

Optical activity was not found in crystals of either of the nonenantiomorphous, uniaxial classes in which Gibbs proved it possible until 1967, when Hobden reported it in silver gallium sulfide, AgGaS$_2$, of point group $D_{2d}(\bar{4}2m)$.[118,150] In the following year he reported it in a second uniaxial crystal, cadmium gallium sulfide, CdGa$_2$S$_4$, of point group $S_4(\bar{4})$.[151] Gibbs's predictions were verified both as to the classes of crystals and the directions in which optical activity is forbidden and permitted. This 85-year lapse between prediction and observation must be one of the longest in scientific history.

E. Why the Long Delay? Linear and Circular Birefringence

Why was there such a long wait? Since the experimental difficulties involve the differences between *linear birefringence* (ordinary double refraction) and *circular birefringence* (circular double refraction,[152] allogyric double refraction[153]), we begin there. Although there is some lack of uniformity in the terminology among physicists, and between physicists and crystallographers, I shall adhere to the terminology of the former, as exemplified by Stone.[154]

1. Linear Birefringence: $|n_o - n_e|$

If we were to measure the refractive index of ordinary glass, using unpolarized monochromatic light,[155] we should find that it has only one index of refraction, no matter what the direction of the incident light ray. The same would be true of sodium chlorate, which belongs to the cubic system, point group $T(23)$.[156] Both glass and sodium chlorate are *optically isotropic*—i.e., the refractive index is independent of the direction of measurement.

Now examine crystalline calcite, $D_{3d}(\bar{3}\ 2/m)$, cut perpendicular to the *direction*[11] in the crystal called the optic axis. When we

pass light through the crystal *normal* to the section, we get only one ray. The index of refraction of the crystal for this ray is called $n_{o(rdinary)}$. However, when light is passed through a section away from the normal, the ray breaks into two, an ordinary ray, for which the medium has an index of refraction n_o, and an extraordinary ray, for which the medium has an index of refraction $n_{e(xtraordinary)}$. Materials which manifest this behavior are said to be *linearly birefringent*, or to show *double refraction*.[157] They are *optically anisotropic*. Typical values of $|n_o - n_e|$ are 10^{-1} to 10^{-3}.[159]

2. Circular Birefringence and Circular Dichroism: $|n_l - n_r|$

Returning to ordinary glass and allowing monochromatic linearly polarized radiation to fall on it, we find the linearly polarized light emerging unchanged.[160] If we were to do this experiment with sodium chlorate, which we found to be optically isotropic like glass when unpolarized monochromatic radiation was used, we should find that the *plane of vibration* has been rotated to the right (clockwise) or to the left (counterclockwise),[161] depending on whether we have (optically) right- or (optically) left-handed sodium chlorate.

The phenomenological explanation for this kind of chiral behavior, as well as for that of the two kinds of α-quartz Biot investigated, goes back to Fresnel in 1825.[9,152] He assumed that *linearly polarized* light is broken within a crystal into two *circularly polarized* rays and proved this with his multiple prism.

An optically active material has one index of refraction, n_r, for *right circularly* polarized light, and another, n_l, for *left circularly* polarized. The possession of two indices of refraction for circularly polarized light is called *circular birefringence*, or *circular double* refraction.[152] A medium which has two such different indices is said to be *optically active*.

A material which shows circular birefringence will also show circular dichroism because of a difference in absorption coefficients k_l and k_r for left and right circularly polarized light.[162] The two phenomena are related in such a way that, if values of one are known as a function of wavelength *over the entire spectral region*, the value of the other may be calculated by using the Krönig–Kramers transform.[162,163] However, aside from references to Foss[112] and to Schellman,[164] who discusses the relation between optical rotation and circular dichroism at a more advanced level, and a mention of linear and circular dichroism later (section III.I), we shall maintain the fiction that materials which are optically active are nonabsorbing, i.e., perfectly transparent.

Since an index of refraction is defined as $n_i \equiv c/v_i$, where c is the velocity of light in a vacuum and v_i its velocity in the medium, Fresnel's hypothesis means that there is a difference in velocity within an optically active medium for the two types of circularly polarized light. From the amount of rotation of the plane of vibration, ϕ, given by[152,159]

$$\phi = \frac{\pi d}{\lambda_0}(n_l - n_r)$$

where d is the thickness of the medium and λ_0 is the wavelength of the light in vacuo, the difference in refractive indices can be calculated.

Now, although the difference in refractive indices, $|n_o - n_e|$, for a *linearly birefringent* material typically amounts to about 10^{-1} to 10^{-3}, the difference in refractive indices for *circularly birefringent* material, $|n_l - n_r|$, generally is of the order of 10^{-4} to 10^{-8}.[159,162] As a result, in a crystal, such as quartz, $D_3(32)$, which has both linear and circular birefringence, the effects of the latter, i.e., of optical activity, may be so swamped by the effects of the former as to be undetectable. In the case of a uniaxial crystal the unwanted effects of linear birefringence are most easily avoided by working, as Biot did (section II.A), with sections
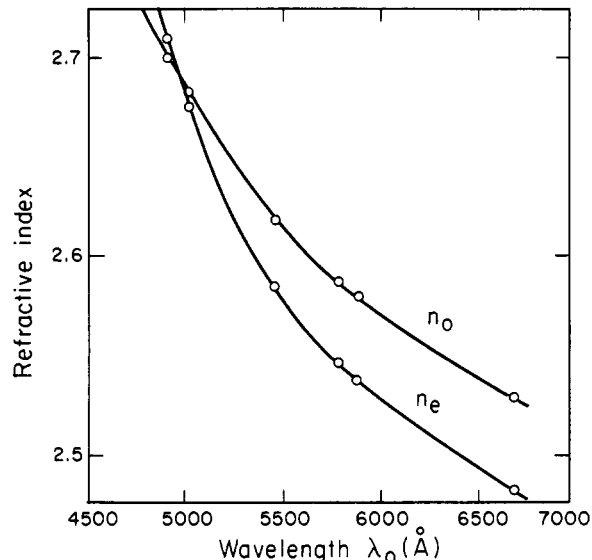

Figure 20. The refractive indices of $AgGaS_2$ at 20 °C (Hobden). Reproduced from ref 150 with permission from *Acta Crystallogr*. Copyright 1968, International Union of Crystallography.

cut perpendicular to the optic axis and having the light incident normal to the section.[165]

3. Off-Axis Elliptical Polarization

When light passes obliquely through the section just described, linear birefringence causes the light to become elliptically polarized, but the effects due to optical activity may be several orders of magnitude smaller.

As mentioned in part A of this section, Gibbs deduced that optical activity along the optic axis is forbidden by symmetry in crystals of classes $D_{2d}(\bar{4}2m)$ and $S_4(\bar{4})$[124]—a conclusion confirmed by Landau and Lifschitz[142] as well as by Nye[159] and many others. Gibbs also said that, for light traveling off-axis, any circular birefringence would probably be very difficult to detect in the presence of elliptical polarization from linear birefringence.

F. Hobden's Experiment

1. Disappearance of Linear Birefringence

How then was Hobden able to detect optical activity in silver gallium sulfide? By the happy accident that at 497.4 nm and 20 °C the indices of refraction for the ordinary and extraordinary rays become equal (Figure 20). Linear birefringence having disappeared, light can be passed through the crystal in any convenient direction to measure circular birefringence.

When Hobden examined single crystals of silver gallium sulfide—point group $D_{2d}(\bar{4}2m)$—cut perpendicular to the optic axis, no optical activity was seen. Along the X and the Y axes, however, optical activity was observed.[150] As Gibbs had predicted, the rotations were equal in magnitude but opposite in sign.

Similar results were obtained with the uniaxial crystals of cadmium gallium sulfide, of point group $S_4(\bar{4})$. Measurement of optical activity in directions away from the optic axis is made possible because at 20 °C the crystal becomes accidentally isotropic for light of wavelength 487.2 nm.[151]

As mentioned previously, Fresnel postulated a difference in velocity for right and left circularly polarized light in right- and left-handed quartz. To account for this physically he also postulated a right-handed helical arrangement of quartz "particles"—the nature of which was unknown at that time—in right-handed quartz, and the opposite helical arrangement in left-handed quartz. A right circularly polarized wave was sup-

posed to travel along a right-handed helix with greater ease, i.e., higher velocity, than a wave of opposite handedness. This helical arrangement was confirmed by X-ray analysis in 1925.[169,170]

Because of the many errors extant, this seems a good place to mention that *dextrorotation* occurs when the tip of the electric vector of a circularly polarized wave traces out a *left-handed* helix. This follows from the current convention for determining the sign of rotation.[161,233]

2. Arrangement of Particles in the Crystal

In a footnote explaining the arrangement of particles in the two nonenantiomorphous classes of crystals which his mathematical analysis had shown could be optically active, Gibbs says:

"There is no difficulty in conceiving of the constitution of a body which would have the properties described above. Thus, we may imagine a body with molecules of a spiral form, of which one-half are right-handed and one-half left-handed, and we may suppose that the motion of electricity is opposed by a less resistance within them than without. If the axes of the right-handed molecules are parallel to the axis of X, and those of the left-handed molecules to the axis of Y, their effects would counterbalance one another when the wavenormal is parallel to the axis of Z."[124]

Hobden notes that prior to Gibbs, "The possibility of spiral structures in crystals with symmetry planes or rotation-inversion axes was overlooked."[150] Describing the structure of silver thiogallate, he speaks of "atomic spirals along the diad axes". Gibbs's description seems to fit the situation in silver thiogallate almost perfectly.

About 85 years after, then, Gibbs's predictions of potential optical activity in two classes of nonenantiomorphous tetragonal crystals were fulfilled.

G. Verification of Pockels' Predictions: Futama and Pepinsky, 1962; Chern and Phillips, 1970

Since Pockels' predictions have to do with optically biaxial crystals, we consider first the conditions necessary for observation of optical activity in class $C_s(m)$. Two cases are possible:

(1) The optical plane (the plane in which the two optic axes lie) may be coincident with the symmetry plane of the crystal (σ, or m). Optical activity along the optic axes is then forbidden by symmetry.[144,171] In other directions optical activity is allowed, but then, as we have seen, the effects of linear birefringence may dominate (section III.E).

(2) The optical plane is perpendicular to the symmetry plane of the crystal. Here rotation of the plane of linearly polarized light is allowed, but the effects must be equal in magnitude and opposite in sign along the optic axes.[144,171] This will be shown in detail later when discussing class $C_{2v}(mm2)$.

Optical activity in a crystal of this kind, the ferroelectric lithium hydrogen selenite, $LiH_3(SeO_3)_2$, was first observed by Futama and Pepinsky,[149] who were also "the first to find the reversibility of the rotatory power when ferroelectric domains switched".[172]

There is some confusion in the literature about optical activity in crystals of class C_{2v}, because it might seem that if one symmetry plane bisects the angle between the optic axes—the condition for seeing optical rotation along the optic axes in class $C_s(m)$—symmetry would require that the optic plane lie in the second symmetry plane of the crystal. Were this so, optical activity along the optic axes would again be forbidden by symmetry.

In Figure 21 general $C_{2v}(mm2)$ symmetry is illustrated, and in Figure 22 the situation in which the optical plane is perpendicular to the principal axis of the crystal, C_2, and therefore to both symmetry planes. The conditions for optical activity here are the same as for case 2 of class $C_s(m)$.

Since reflection reverses handedness (Figure 16), it is clear from Figure 22 that the left-rotating axis will be reflected into
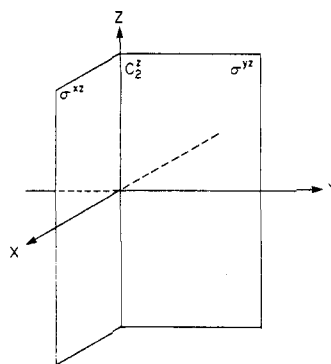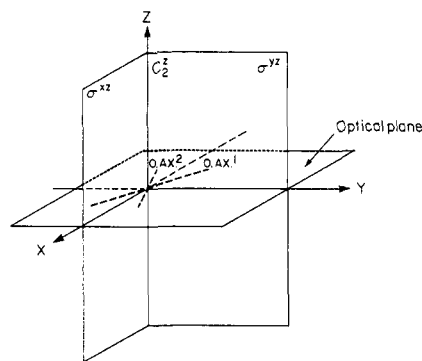

Figure 21. General $C_{2v}(2mm)$ symmetry.

Figure 22. Crystal of C_{2v} symmetry; optical plane perpendicular to C_2 axis.
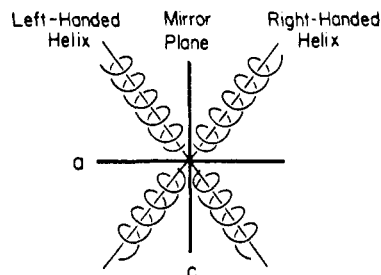

Figure 23. Change of handedness of helixes upon reflection in a mirror plane (from Chern and Phillips).[174] Reprinted with permission from ref 174. Copyright 1972, American Institute of Physics.

a right-rotating one, and vice versa; and dextrorotated light will be converted into levorotated light, and vice versa, by the symmetry operations of the crystal.

Chern and Phillips were the first to observe optical activity along the optic axes of a crystal of this kind, a ferroelectric sodium nitrite crystal grown from the aqueous phase.[173] In a later paper they investigated the relation between optical rotation and spontaneous polarization as a function of temperature. They found, as had Futama and Pepinsky,[149] reversal of the sense of optical rotation when a single-domain crystal was switched by a dc electric field and pointed out that optical rotation can be used to determine domain structure in ferroelectric crystals.[174] Figure 23, taken from this second paper, illustrates the relation of right and left helical arrangement of the particles of such a crystal in relation to its symmetry elements.

Rotation of the plane of linearly polarized light has now been observed in all the classes predicted by Gibbs and Pockels. Since the fortuitous occurrence of the disappearance of linear birefringence (section III.F) is probably rare, future progress will lie in being able to measure circular birefringence in the presence of linear birefringence (section III.I). Before this is taken up, the necessary conditions for optical activity in crystals and the classes of crystals in which optical activity is allowed will be summarized.

H. Necessary Conditions for Optical Activity in Crystals: Crystal Classes in Which Optical Activity Is Possible

Of the 32 crystal classes, 11 may show optical activity because, possessing only axes of symmetry, they are enantiomorphous. Of the remaining 21 nonenantiomorphous classes, we have seen not only that current theory says that crystals of 4, $C_s(m)$, $C_{2v}(2mm)$, $D_{2d}(\bar{4}2m)$, and $S_4(\bar{4})$, may be optically active, but that this has been verified.

In 1903–1904, about 8 years after Pasteur's death, Voigt and Chipart showed independently that a necessary, but not a sufficient, condition for optical activity in crystals is the absence of a center of symmetry.[144,175–180] The method involves showing that all nine components of the optical gyration tensor are identically equal to zero if a center of symmetry is present.[144] Optical activity is thus impossible in the 11 nonenantiomorphous classes of crystals having a center of symmetry.

Fedorov's analysis, following a new route,[131,134,171] shows that the presence of an $S_3(\bar{6})$ axis prevents activity in crystals. The absence of an $S_3(\bar{6})$ axis is a second necessary, but not sufficient, condition for optical activity in crystals. This eliminates the nonenantiomorphous classes $C_{3h}(\bar{6}$, or $3/m)$ and $D_{3h}(\bar{6}2m)$. It was shown previously (section II.E.1.c and Table II) that the i operation produces the same effect as the S_2.[102] Therefore the necessary, but not sufficient, conditions of this paragraph and the preceding one can be combined by saying that the *absence* of both the S_2 and S_3 axes is required for optical activity.

Of the remaining four nonenantiomorphous classes, it is universally agreed that crystals belonging to $T_d(\bar{4}3m)$ must be inactive because, like crystals having a center of symmetry, all of the elements of their optical gyration tensor are identically equal to zero.[135,137,177,178,181,182]

For *ordinary* optical activity (rotation of the plane of linearly polarized light) we anticipate some future results (section V.C) to say that a third necessary, but not sufficient, condition for optical activity in crystals is the *absence* of a C_n axis ($n \geq 3$) lying in a plane of symmetry.[131,171] This provides another explanation for the optical inactivity of crystals of point group $T_d(\bar{4}3m)$, since each of the four threefold axes lies in a plane of symmetry.

Crystals of nonenantiomorphous classes $C_{3v}(3m)$, $C_{4v}(4mm)$, and $C_{6v}(6mm)$ cannot rotate the plane of linearly polarized light in any direction but do manifest the third kind of optical activity: the polar longitudinal effect[122,131–133] (Appendix V.C). There are thus 18 classes of crystals in which some kind of optical activity is possible.[183]

I. Detection of Circular Birefringence in the Presence of Linear Birefringence

Because circular birefringence is often several orders of magnitude less than linear birefringence (section III.E), it would be most useful if methods were developed for measuring circular birefringence in the presence of linear birefringence. This is especially so since optical activity can be used to give information on the nature of bonding in crystals.[172] Kobayashi and co-workers, for example, have extended Born's theory of optical activity[184] to apply it to ferroelectricity.[185]

"Simultaneous Measurement of Linear and Circular Birefringence in Crystals" was reported by Yu and Barker at the annual meeting of the Optical Society of America in 1975.[186] The method involves measuring changes in elliptically polarized light as it passes through the sample. Electrooptic measurements were made on bismuth silicon oxide and bismuth germanium oxide—both point group $T(23)$. These were chosen because, in addition to the natural optical activity (circular birefringence) which these enantiomorphous crystals have, additional linear

and circular birefringence could be induced by an electric field.

The most extensive description of a method for measuring optical activity in crystals away from the optic axes is that of Kobayashi and co-workers.[172] The principle is to measure accurately the *intensity* of light emerging from a properly oriented sample. By taking the ratio of the intensities with the sample alternately between parallel and crossed polarizers, both the circular birefringence, $n_o - n_e$, and components of the optical gyration tensor (Appendix V.C.2) can be determined as a function of wavelength.

These authors found that the paraelectric phase, symmetry point group $D_{2d}(\bar{4}2m)$, of KH_2PO_4 (KDP) is optically active and measured its gyration coefficient. As explained before (section III.A,D,E), this represents a special challenge, since optical activity in such a uniaxial crystal is forbidden by symmetry along the optic axis.

In a subsequent publication Uesu and co-workers improved their apparatus so that the accuracy of the measurements of the elements of the optical gyration tensor is "two orders of magnitude better than that of the best previous one".[187] This sensitivity enabled them to report the first observation of the electrogyration effect of a nonenantiomorphous crystal, KDP, and to obtain the electrogyration coefficient of this ferroelectric over a wide range of temperature, including the transition point.

Lest the emphasis in this section give the impression that measurement of circular birefringence in the presence of linear birefringence is of interest only in solid crystals, mention should be made of its measurement in oriented biopolymers by Hofrichter and Schellman, who have devised instrumentation which can also measure both circular and linear dichroism.[188] In their introduction they mention several methods which have been used to measure the optical properties of polymers, together with several suggestions as to the kind of information which might be derived by studying the optical properties of oriented biopolymers.

In addition, linear and circular dichroism in J-aggregates of dyes have been studied by Norden,[189] as well as by Saeva and co-workers.[190,191]

IV. Optical Activity in Nematic Liquid Crystals

Although the discussion here is limited to nematic liquid crystals, it will become clear that the conclusions of Gibbs and Pockels with respect to optical activity in nonenantiomorphous crystals will be valid for any mesophase with sufficient order and periodicity when the oriented domains cooperate with one another.

We have seen above (section II.A and introduction to section III) that the early observation that molecular enantiomorphism is the necessary condition for optical activity for small molecules in the isotropic liquid phase was coupled with some mistaken views as to the relation between molecular enantiomorphism and crystal structure. This led to the erroneous conclusion that enantiomorphism of configuration is essential to optical activity in crystals.

Similarly, although early researches by Reinitzer on cholesterol and its derivatives, substances which are dissymmetric (chiral) in the mesomorphic (liquid crystal) state of matter, led to the discovery of optical activity,[192] it also led, unfortunately, to the conclusion that, "This phenomenon is always associated with the presence of dissymmetric molecules."[193] The explanation of optical activity in nonenantiomorphous liquid crystals is thus an integral part of the demonstration that enantiomorphism of configuration is not always essential for optical activity.

The optical activity of nematic liquid crystals referred to here is not that brought about by the formation of a helical, cholesteric phase from a nematic phase by the addition of nonmesomorphic molecules.[194,195] Nor is it the optical activity of a twisted nematic phase, e.g., between rubbed surfaces.[190] Instead, it is that due
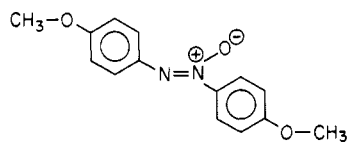
Figure 24. *p*-Azoxyanisole, point group C_s.

to an optically active nematic mesophase composed of non-dissymmetric (achiral) molecules, *p*-azoxyanisole (PAA) (Figure 24).

A. Williams' Experiment

Richard Williams has observed optical rotation and a linear electrooptical effect in the nematic liquid phase of PAA.[196,197] Since the molecule has a plane of symmetry, point group C_s, it cannot be dissymmetric (chiral) and is therefore optically inactive in the isotropic liquid phase. Williams concluded from his experiments that the *nematic* liquid crystal *phase* of PAA has, like the individual molecules in the isotropic liquid phase, a plane of symmetry.[197]

It was suggested that the rotation observed with PAA might be due to changes of orientation occurring between the surface layer and the interior of the liquid.[198] Although subsequent experiments on nematic liquid crystal films of *N*-(4-methoxy-benzylidene)-4-*n*-butylaniline (MBBA) revealed that, for this compound, a thin layer near the surface dominated the optical rotation, in the case of PAA the fact that the rotation varied with the thickness of the sample indicates that, for PAA, this is a bulk effect.[199]

B. Kimura's Theory

Because many liquid crystal systems have *molecules* with C_s and C_{2v} symmetry, Kimura investigated theoretically the behavior of liquid crystals with C_s and C_{2v} symmetries—the same two classes which Pockels had predicted would allow optical activity in nonenantiomorphous crystals (section III.B).[200] Using the results of a previous paper with Nakano,[201] he derived an optical activity coefficient which characterizes the optical activity of a *liquid crystal phase* composed of molecules having C_s or C_{2v} symmetries:

$$\beta = -\beta_1 \langle \sin 2\theta \sin \psi \rangle + \beta_2 \langle \sin 2\theta \cos \psi \rangle$$

In this expression θ and ψ are two of the three Eulerian angles relating the molecular coordinate frame to the laboratory coordinate frame. In isotropic liquids the averages over all the random orientations of θ and ψ cause each of the bracketed expressions to be zero. This makes the optical activity coefficient zero, in agreement with previous conclusions (section II.E.2).

If, however, all of the molecules have a unique orientation, β will be proportional to $\sin 2\theta$, where θ is the angle between the molecular ζ axis and the direction of propagation of the light (Z axis). Kimura concludes that for light traveling *normal* to the plane of a molecule of C_s or C_{2v} symmetry ($\theta = 0$ or π) or *in* the mirror plane of the molecule ($\theta = \pm\pi/2$), optical activity will be absent, but for all other angles optical activity can, in principle, be present if the molecules form a regular lattice.

His final conclusions are that, for properly oriented molecules, optical activity can be present in a nematic liquid crystal phase composed of molecules of either C_s or C_{2v} symmetry. However, only the former can exhibit the linear electrooptic effect found by Williams. Kimura also agrees with Williams that the *nematic* liquid crystal *phase* of PAA has a plane of symmetry, point group C_s.

This is an example of the second factor given by Jerephagnon and Chemla,[122] which was mentioned in the introduction to section III: structural rotatory power—a rotation of the plane

of linearly polarized light by a substance which may be optically inactive in the isotropic liquid phase.

V. Appendixes

A. Differences between van't Hoff and Le Bel

There are two minor and two major differences. van't Hoff's thought was a continuation of Kekulé's concept of the quadrivalence of carbon, whereas Le Bel's was partly a more precise specification of Pasteur's idea of a dissymmetric tetrahedral arrangement (section II.A).[202] Le Bel's treatment was more general, van't Hoff's more specialized, as van't Hoff himself admitted.[120]

The major points of disagreement, as summarized by Sementsov,[36] are the differing characters of the asymmetric carbon atom which they hypothesized and differences in the nature of ethylene and its derivatives.

1. Le Bel's Denial of a Tetrahedral Hypothesis

Although the previous discussion (section II.B) makes it clear that van't Hoff hypothesized a tetrahedral arrangement of groups as constituting an asymmetric carbon "atom", this was not true for Le Bel. In fact the latter denied several times that he hypothesized a tetrahedral arrangement,[203] although he recognized this as a possibility in special cases.[24] Since I can hardly improve on the translation of Le Bel's own words, given by Sementsov, and taken from the first two paragraphs of ref 203, I quote it:

"I used the greatest efforts in all my explanations to abstain from basing my ideas on the preliminary hypothesis that the compounds of carbon of the formula CR_4 have the shape of a regular tetrahedron.

"It happened that very many scientists, who wrote about my article most favorably in other respects, did not turn their attention to the fundamental difference between my starting point and the starting point of M. van't Hoff in his analogous work published at the same time in Utrecht."

2. Representation of Ethylene and Its Derivatives

van't Hoff represented a C=C bond by two tetrahedra joined along an edge, as discussed in connection with allene derivatives (section II.B). This requires that ethylene derivatives be planar, as can be seen from Figure 3: drop out the lowermost carbon and attach R_3 and R_4 to the remaining one. If ethylene derivatives are planar, the isomerism of maleic and fumaric acids is easily explained, and the proper number of isomers for ethylene derivatives is obtained.[204,205]

Le Bel, however, tried several times to resolve into optical isomers methylmaleic (citraconic) and methylfumaric (mesaconic) acids, as well as compounds of the type $C(R)_2R_1R_2$.[206] According to his hypothesis of a squarish pyramid with carbon at the apex, which was much more in accordance with the theories of valence held by physicists at that time[203] than was the tetrahedral hypothesis of van't Hoff,[36] there should be a racemic pair of cis isomers and an optically inactive trans isomer for $CRRR_1R_2$. He did not admit the planarity of ethylene derivatives until 1882.[207]

B. Sommerfeldt's Crystals

These crystals are commonly referred to as the dimer of the β form of methyl mesityl oxide oxalate, or by the German name bis(β-methylmesityloxydoxalat),[208,209] $(C_9H_{12}O_4)_2$, with the structural formula given in section III.C. The systematic name would be bis(methyl 6-methyl-2,4-dioxo-5-heptenoate). However, there is some question as to the identity of the material. The descriptions of the preparation of both the α and β forms of it in Beilstein,[210] following Federlin,[211] refer to obtaining the desired

compound after it has stood several days in *diffuse daylight*. It seems probable that photodimerization occurred, perhaps with formation of a cyclobutane ring.

The syntheses reported most recently[173,212] have been aimed principally at reproducing the crystals which Federlin made, whose crystal habit and optical activity were reported by Sommerfeldt (section III.C). Following each step of the synthesis with modern methods should establish the chemical identity of each of the two α and two β dimers.[211]

Rogers, having made "crystals corresponding exactly with those described by Sommerfeldt", showed by X-ray analysis that they belong to space group $P2_1/c$,[212] point group $2/m$.[173] Since this has a center of symmetry, the crystals must be optically inactive (section III.H). Like Voigt,[147] Rogers believes that Sommerfeldt's optical observations were caused by twinning in his crystals, and explains how Sommerfeldt may have been misled. The controversy has been reviewed, in considerable detail, by Tatarskii,[171] who was unaware of Rogers' work.

A reexamination of the ability of the crystals to rotate the plane of linearly polarized light along the optic axes was made by Chern and Phillips, who found none.[173] The first report, then, of optical activity in a nonenantiomorphous crystal seems to have been made by Futama and Pepinsky.[149]

C. Optical Activity in Crystals of Classes $C_{3v}(3m)$, $C_{4v}(4mm)$, and $C_{6v}(6mm)$: W. Voigt and F. I. Fedorov

1. W. Voigt: Elliptically Polarized Reflected Light

In two papers,[177,213] especially the second in 1905, Voigt said that rotation of the plane of linearly polarized light would not be observed along the optic axis of crystals of these classes. He believed, however, that linearly polarized light, incident normally on a section cut perpendicular to the optic axis, would be reflected as elliptically polarized. Voigt thought that his failure to observe this with crystals of tourmaline[171] could be due to the obscuring of small differences by surface defects in his crystals.[213] We shall see later that the axial ratio of the elliptically polarized light is expected to be less than 10^{-3}.

For all three of these classes, Voigt's analysis showed that seven of the nine elements of the optical gyration tensor were identically equal to zero, but that two of them were antisymmetrically related: $g_{21} = -g_{12}$.[181,182,213,214]

2. The Optical Gyration Tensor and the Scalar Parameter of Gyration

a. Pre-Fedorov. Because optical activity in crystals is still a frontier problem in crystal physics,[154] which has led to many errors in the literature,[136] different forms of the optical gyration tensor and the related scalar parameter of gyration have been used. The outline here, following Born and Nye, will give some idea of the difficulties.

The specific rotatory power, i.e., the rotation per unit path (for crystals, usually 1 mm), is[136,215]

$$\rho = \frac{\pi G}{\lambda_0 \bar{n}}$$

where G is the scalar parameter of gyration,[216] $\bar{n}$ is the refractive index along the optic axis, and λ_0 is the wavelength of the light in vacuo.

Using a simplification introduced by Voigt, Born derives the equation for G in quadratic form:

$$G = g_{11}l_1^2 + g_{22}l_2^2 + g_{33}l_3^2 + \bar{g}_{12}l_1l_2 + \bar{g}_{13}l_1l_3 + \bar{g}_{23}l_2l_3$$

where $\bar{g}_{ij} = \frac{1}{2}(g_{ij} + g_{ji})$.[216] The l_i are the direction cosines of the wave normal with respect to axes arbitrarily chosen in the crystal,[136] and the g_{ij} are the elements of the optical gyration

tensor. Jones[134] discusses the error which Born makes in calling this a "material" tensor.[217]

Because the optical gyration tensor, which is generally not symmetric,[220] may sometimes be antisymmetric,[181,182,213,214] it is possible to have $G = 0$, even when not all the elements of the optical gyration tensor are identically equal to zero. This is the case for the three classes of crystals under discussion, since $g_{12} = -g_{21}$, and all other elements are zero. The specific rotatory power being zero, there will be, as Voigt said, no rotation of the plane of linearly polarized light along the optic axis. It is for this reason that crystals of these three classes were previously considered to be optically inactive.[132]

b. Fedorov: *Extension of the Definition of Optical Activity.* Fedorov, as Jones has pointed out (see note in ref 131), was the first to get the correct optical gyration tensor for an anisotropic nonabsorbing crystal. The constitutive relations, $\vec{D} = \epsilon\vec{E}$ and $\vec{B} = \mu\vec{H}$, for dielectric materials must be modified if the materials are optically active. Although the contribution from the magnetic field is usually estimated to be of the same magnitude as that from the electric field,[182,221] it was customary, prior to Fedorov, to neglect the magnetic effect because of mathematical complexities.[131,132,182,216,221]

Fedorov, however, making use of the constitutive relations

$$\vec{D} = \epsilon\vec{E} + \alpha\vec{\nabla} \times \vec{E}$$

$$\vec{B} = \mu\vec{H} + \beta\vec{\nabla} \times \vec{H}$$

where α and β are the electric activity and magnetic activity tensors, assuming no necessary relation between α and β,[222] taking precise account of the law of conservation of energy, and using an approach which did not restrict him to any particular coordinate system,[131,225] succeeded in getting the electric and magnetic activity tensors for all 32 crystal classes.

For the three classes of crystals under consideration, Fedorov found that the electric and magnetic activity tensors are antisymmetric[131,132] and showed that linearly polarized light, when reflected or transmitted under proper conditions by these crystals,[132,133] should be elliptically polarized. He concluded that a proper definition of optical activity is one for which his constitutive relations hold, with at least one of α or β not being identically equal to zero. After a brief mention of the approach used by Jerephagnon and Chemla,[122] this section will be concluded by reference to the experiment which Fedorov and co-workers have proposed to detect the change from linearly to elliptically polarized light upon reflection.

c. Post-Fedorov. Other approaches to determining whether a material can be optically active are possible, but only that of Jerephagnon and Chemla is mentioned because of an estimate they give as to the magnitude of the polar longitudinal effect, which has a direct bearing upon Fedorov's proposed experiment.[122]

Before going on to prepare an alternative formulation of optical activity in terms of specific rotativity, Jerephagnon and Chemla decompose their optical activity tensor into three irreducible, independent tensors, which they call a pseudoscalar, a vector, and a pseudodeviator.[122] If, for any crystal class, one or more of these are not identically equal to zero, members of that class may be optically active.

Their table for the irreducible components of the optical activity tensor for each of the 21 classes of crystals lacking a center of symmetry shows that crystals of classes $C_{3v}(3m)$, $C_{4v}(4mm)$, and $C_{6v}(6mm)$ are unique in having only a "vector component" not equal to zero. Such crystals can produce only a polar longitudinal effect. This means that, for these classes, light which is linearly polarized along the Z axis, but traveling along the X axis, should produce an out-of-phase component, E_X. In the conclusion of this section, only the E_Y component, which is produced by linearly polarized light traveling along both the X and Y axes, is shown.
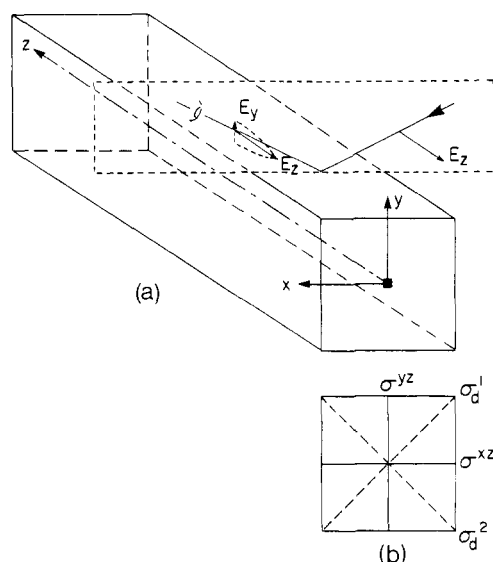
(a)

(b)

Figure 25. Elliptically polarized light produced by reflection of linearly polarized light (after Fedorov et al.[227] and Tatarskii[171]).

For light polarized in the Z direction and traveling along the X axis, Jerephagnon and Chemla estimated the ratio of E_X/E_Z to be about $(a/\lambda) \simeq 10^{-3}$,[226] a being a "typical interatomic distance and λ the optical wavelength". They ascribed the apparent failure to observe the appearance of elliptically polarized light, resulting from the polar longitudinal effect, to the smallness of this ratio.

3. Detection of Elliptically Polarized Light upon Reflection from Crystals of These Classes

Fedorov and co-workers[227] have proposed an experiment to show that the reflection of linearly polarized light produces elliptically polarized light in the classes under discussion. The principal details of the experiment may be summarized as follows:[171,227] (a) The reflecting plane is the surface of a prism [trigonal (C_{3v}), tetragonal (C_{4v}), or hexagonal (C_{6v})], which is perpendicular to one of the symmetry planes of the crystal. (b) The plane of incidence of the linearly polarized beam is perpendicular to the surface of the prism and to the optic axis. (c) The electric vector of the linearly polarized light is perpendicular to the plane of incidence[228] and parallel to the optic axis, and it lies in a symmetry plane of the crystal.

All of these features are embodied in Figure 25a, where, for simplicity of construction, a crystal of one of the tetragonal classes, $C_{4v}(4mm)$, has been chosen. Such crystals have a unit cell with $\alpha = \beta = \gamma = 90°$, and axes $a = b \neq c$.[231] In the figure, the optic axis—the c axis—which is a fourfold symmetry axis, coincides with the Z axis. In Figure 25b the two diagonal planes of symmetry, σ_d, which must be present together with the two vertical planes of symmetry, σ_v^{XY} and σ_v^{YZ} for $C_{4v}(4mm)$, are shown.[232]

If no elliptical polarization occurs, the reflected beam will simply be linearly polarized. However, if elliptical polarization occurs, the polar longitudinal effect will produce an out-of-plane E_Y component (shown in Figure 25a), which will show up as right- or left-elliptical polarization.[233]

Fedorov, Bokut, and Konstantinova note particularly that "neither complete reflection from a transparent crystal nor reflection from an absorbing crystal [i.e., a uniaxial, optically inactive crystal[132]] under the same conditions [optic axis parallel to the electric vector so that the electric vector of the linearly polarized light vibrates in a symmetry plane of the crystal] can lead to this effect". . . "Consequently, in this case, elliptical polarization of the reflected wave is caused exclusively by optical activity, and is a sign of the presence of the latter."[227]
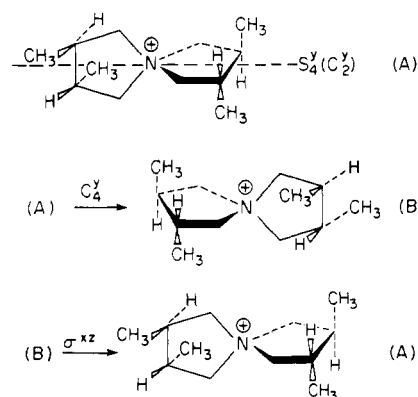

Figure 26. The S_4 axis and the complex S_4 operation.

Because the effect expected is ordinarily extremely small, Fedorov and co-workers make additional suggestions for doing the experiment. The crystal should be placed in a cuvette and covered with a liquid. The linearly polarized light is incident upon the liquid–crystal interface through this liquid, and the reflected beam is observed through it also. By dropwise addition of a second liquid, the index of refraction, n, of the mixture is brought as close as necessary to n_e of the crystal (section III.E). When $n - n_e$ is made sufficiently small, the ellipticity of the reflected wave can be considerably increased. For additional details the original article should be consulted.[227]

It is not clear whether these authors have actually tried the experiment. As mentioned at the end of section V.C.2, the anticipated effect does not so far seem to have been detected.

D. The S_4 Axis

For completeness, an explanation of this less familiar element is included. Although the S_4 axis is chosen for concreteness, the explanation applies to the S_n axis of any order if C_n is substituted for C_4.[102,105]

This symmetry element is known as an alternating axis of symmetry, rotation–reflection axis, improper axis of rotation, and mirror axis. It is a symmetry element of the second kind. It is related to the rotation–inversion axis used by crystallographers. We approach the element through the corresponding symmetry operation.

Consider the tetramethylspirobipyrrolidinium cation (Figure 26), made specifically to test the prediction that an entity with S_4 and C_2 axes as its only true symmetry elements would be optically inactive (section II.E.1.d).

With the convention for the direction of axes of Figure 19, the dashed axis of Figure 26A is a Y axis. *Looking along it to the left*, toward the XZ plane, do a C_4^Y operation, a 90° clockwise rotation, *even though the cation does not have a* C_4^Y *axis*. Then

$$(A) \xrightarrow{C_4^Y} (B)$$
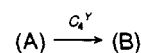
In testing for the presence of an element of symmetry other than an S axis, the observer must not be able to distinguish the conformation of the object after the operation has been performed from its initial conformation. Because the S operation is a complex, two-step operation, this criterion does not apply until after the second step has been completed.

Reflecting (B) in the XZ plane, *even though the cation does not have* σ^{XZ}, yields a conformation indistinguishable from the original:

$$(B) \xrightarrow{\sigma^{XZ}} (A)$$
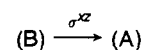
This shows that $\sigma^{XZ}C_4^Y = S_4^Y$, the order of the operations on the left-hand side being read from right to left. However,
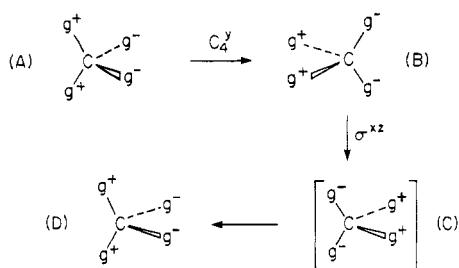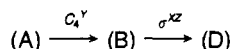
Figure 27. Application of S_4 operation to the molecule of Figure 14D.

It can be shown that $C_4{}^Y \sigma^{XZ} = S_4{}^Y$; i.e., the order of the operations is immaterial.[7,8a]

The application of the S_4 operation to the molecule of Figure 14D is shown in Figure 27. The actual series is

$$(A) \xrightarrow{C_4{}^Y} (B) \xrightarrow{\sigma^{XZ}} (D)$$

Since (D) is indistinguishable from (A), the molecule has an $S_4{}^Y$ axis coincident with the $C_2{}^Y$ axis.

Because reflection—or indeed any S operation in the broad sense, as the term is used in Table II—reverses handedness simultaneously, the σ^{XZ} operation upon (B) straightway produces (D). In Figure 27 an additional step (C) and a bracketed intermediate form have been used to make the reflection and reversal a two-step process to assist in picturing the steps.

That reflection reverses handedness (chirality) is illustrated in Figure 16 with the chiral *sec*-butyl group.

VI. References and Notes

(1) Jaeger, F. M. "Lectures on the Principle of Symmetry and Its Applications in All Natural Sciences"; Elsevier: Amsterdam, 1917.
(2) Jaeger, F. M. "Le Principe de Symétrie et Ses Applications"; Gauthier-Villars: Paris, 1925. A translation from the second English edition of ref 1, 1920.
(3) Freudenberg, K., Ed.; "Stereochemie"; F. Deuticke: Leipzig und Wien, 1933.
(4) Pasteur, L. "Leçons de Chimie"; Lib. Hachette: Paris, 1861; p 24. This mature account of Pasteur's discovery of *l*-tartaric acid, and of the demonstration which he was required to make to prove this before J. B. Biot, is one of the most illuminating and touching in the history of science. Besides being a fine account of the whole development of

the causes and occasions of optical activity, it is also an excellent antidote to the colorless and rather misleading accounts of the nature of science and/or the scientific method which are sometimes found in the early chapters of elementary textbooks of science.
(5) Pope, Sir W. J. *Bull. Soc. Chim. Fr.* **1919**, *25*, 427–445. Sir William's fine article, in French, on molecular dissymmetry is marred by a number of errors with respect to optical activity in crystals. He was apparently unaware of the work of Gibbs (1882) and of Pockels (1906) to which reference is made in section III. Also, he was confused as to the essential condition for enantiomorphism. On p 439 he says it is the absence of a center of symmetry, whereas it is actually the absence of the same three elements which must be *missing* if there is to be optical activity, as explained in section II.E. However, in all of his papers he seems to have been perfectly clear as to the meaning of dissymmetry. Never once did he make the error, so common among his English, German, and American contemporaries, of confusing it with asymmetry.
(6) Lowry, T. M. "Optical Rotatory Power"; Dover: New York, 1964 (reprint of 1935 ed.), pp 25–26.
(7) Jaffé, H. H.; Orchin, M. "Symmetry in Chemistry"; Wiley: New York, 1965; Chapter 2.
(8) (a) Cotton, F. A. "Chemical Applications of Group Theory", 2nd ed.; Wiley-Interscience: New York, 1971; Chapter 3. (b) Rinne, F. "Crystals and the Fine-Structure of Matter"; Dutton: New York, 1924; p 27. (c) Klug, H. P.; Alexander, L. E. "X-Ray Diffraction Procedures", 2nd ed.; Wiley: New York, 1974; p 22.
(9) Reference 6, general. Lowry's book is a good general source for precise references to the works of Arago, Biot, Fresnel, and others who developed the theory of optical activity. Very often Lowry quotes so extensively that consulting the original text is unnecessary. See ref 10, however.
(10) So many errors are spread throughout the literature, because the original texts of J. H. van't Hoff, J. A. Le Bel, and L. Pasteur have not been consulted, that the reader fluent in French and German would do well to check the originals. A specific example of erroneous editorial addition by a translator is given in section II.D.
(11) Direction is emphasized because, contrary to what the term suggests, the optic axis is not a unique axis, but a direction in the crystal.
(12) (a) Herschel, J. F. W. *Trans. Camb. Phil. Soc.* **1822**, *1*, 43–52. (b) Wahlstrom, E. E. "Optical Crystallography", 5th ed.; Wiley: New York, 1979; pp 365–366, 370–372. (c) Reference 145, pp 133–135. (d) Reference 119, pp 169–170.
(13) Kauffman, G. B.; Myers, R. D. *J. Chem. Educ.* **1975**, *52*, 777–781.
(14) Fieser, L. F.; Fieser, M. "Advanced Organic Chemistry"; Reinhold: New York, 1961; pp 69–71.
(15) Pasteur, L. *Ann. Chim. Phys.* **1848**, *24*, 442–459.
(16) Kauffman and Myers[13] note that in Pasteur's time what is now called "racemic tartaric acid" was simply known as "racemic acid" from Latin *racemus*, a cluster of grapes.
(17) Since an *asymmetric* object is never superposable on its mirror image, and all asymmetric objects are therefore dissymmetric, it may seem that the terms molecular *dissymmetry* and *dissymmetric* are superfluous. Indeed *asymmetric* and *dissymmetric* are frequently confused both in dictionaries (English and German) and in the literature. Since, however, a *highly symmetric* object might not be superposable on its mirror image, two different categories are involved. We shall see later that objects may be asymmetric dissymmetric, symmetric dissymmetric, and symmetric nondissymmetric.
(18) Condon, E. U. *Rev. Mod. Phys.* **1937**, *9*, 432–457.
(19) Kauzmann, W.; Walter, J. E.; Eyring, H. *Chem. Rev.* **1940**, *26*, 339–407.
(20) Kauzmann, W. "Quantum Chemistry"; Academic Press: New York, 1957; Chapters 15 and 16.
(21) Schellman, J. A. *Acc. Chem. Res.* **1968**, *1*, 144–151.
(22) Caldwell, D. J.; Eyring, H. "Theory of Optical Activity"; Wiley: New York, 1971.
(23) Mann, F. G.; Pope, W. J. *J. Soc. Chem. Ind. London* **1925**, *44*, 833–835.
(24) Le Bel, J. A. *Bull. Soc. Chim. Paris* **1874**, *22*, 337–347.
(25) Pasteur, L. "Researches on Molecular Asymmetry of Natural Organic Products"; Alembic Club Reprint No. 14, The Alembic Club: Edinburgh; University of Chicago Press: Chicago, 1906.
(26) Reference 6, p 27, footnote.
(27) Werner, A. *Bull. Soc. Chim. Fr.* **1912**, *11*, I–XXIV. Since Professor Werner apparently sent in his contribution too late for regular pagination, these pages are bound in diverse places by different bookbinders. There are at least three sets with Roman numerals in this volume.
(28) *Chem. Zentralbl.* **1912**, *83*, Part 2, 798. Since the German abstractor did not recognize the honorific French M. (Monsieur), this article is referred to in author indexes, for example in *Chem. Abstr.*, under Werner, M. A., instead of Werner, A.
(29) Werner, Alfred. *Am. Chem. J.* **1913**, *49*, 314–336. This reference is correct. The one given in *Chem. Abstr.* **1913**, *7*, 1333, which refers to Vol. 48, is wrong.
(30) van't Hoff, J. H. "Voorstel tot Uitbreiding---" (---: 29 more words), J. Greven, Utrecht, 1874. Translated by G. M. Richardson, in "The Foundations of Stereo Chemistry"; memoirs by Pasteur, van't Hoff, Le Bel, and Wislicenus; Richardson, G. M., Ed.; American Book Co.: New York, 1901; pp 35–46.
(31) van't Hoff, J. H. *Arch. Neérl. Sci. Exactes Natur.* **1874**, *9*, 445–454. This is essentially a translation of the "Voorstel".
(32) van't Hoff, J. H. *Bull. Soc. Chim. Paris* **1875**, *23*, 295–301. This paper has significant advances over the article in *Arch. Neérl.*
(33) Prior to van't Hoff, others had suggested a tetrahedral arrangement for the carbon atom. Lowry[34] refers to Paterno (1869), whose good idea in this respect failed when it was shown that three rigid isomeric

forms of dichloroethane did not exist. In "Forty Years of Stereochemistry"[35] Pope mentions that W. H. Wollaston (1808) suggested that a stable arrangement for AB_4 would be obtained "if the four [B] particles are situated at the angles of the four equilateral triangles composing a regular tetrahedron".

(34) Reference 6, pp 38–39.
(35) Pope, Sir W. J. *J. Soc. Chem. Ind. London* **1932**, *51*, 229–233T.
(36) Sementsov, Anatol. *Am. Sci.* **1955**, *43*, 97–100.
(37) A resumé of the differences between the thoughts of van't Hoff and Le Bel is given in Appendix V.A.
(38) Snelders, H. A. M. *J. Chem. Educ.* **1974**, *51*, 2–7.
(39) van't Hoff, J. H. "La Chimie dans l'Espace"; P. M. Bazendijk: Rotterdam, 1875; 44 pp + 3 plates, p 8.
(40) van't Hoff, J. H. "Die Lagerung der Atome in Raume"; F. Vieweg und Sohn: Braunschweig, 1877; p 5 (an excellent translation of Dr. F. Herrmann, who corrected some of van't Hoff's errors, and seems to have made almost none of his own).
(41) Reference 39, pp 6–8, 15–16.
(42) Reference 40, pp 2–4, 16–18.
(43) Hirschmann, H.; Hanson, K. R. *J. Org. Chem.* **1971**, *36*, 3293–3306.
(44) Reference 39, pp 29–30, Plate II, Figures XXIX–XXXI. Because ref 30–32 are frequently, and erroneously, cited for van't Hoff's allene derivatives, it is emphasized again that allene derivatives are first mentioned in ref 39.
(45) Reference 40, pp 14, 44–45, 49–50, 53.
(46) Instituts Solvay, Brussels, "Institut International de Chimie"; Premier Conseil de Chimie, 1922; Gauthier-Villars: Paris, 1925; Sir Wm. Pope, pp 141–175; discussions by F. M. Jaeger, pp 160–168.
(47) Jaeger, F. M. *Bull. Soc. Chim. Fr.* **1923**, *33*, 853–889.
(48) Reference 1, Chapter 5.
(49) Jaeger, F. M. "Spatial Arrangements of Atomic Systems and Optical Activity"; McGraw-Hill: New York, 1930; pp 46, 60. This book is sometimes referred to as "Optical Activity and High Temperature Systems".
(50) Mislow, K. "Introduction to Stereochemistry"; W. A. Benjamin: Menlo Park, CA, 1966; pp 27, 32, 85–86, 167.
(51) Prelog, V. *Proc. Kon. Ned. Akad. Wetensch.* **1968**, *B71*, 108–123.
(52) Aschan, Ossian, *Chem. Ber.* **1902**, *35*, 3389–3399, at p 3396. On the first page of this article Aschan states incorrectly that Pasteur ascribed possible optical activity to asymmetry of a molecule.
(53) Reference 47, pp 864–867.
(54) Reference 6, pp 62–67.
(55) Maitland, P.; Mills, W. H. *J. Chem. Soc.* **1936**, 987–998.
(56) Kohler, E. P.; Walker, J. T.; Tishler, M. *J. Am. Chem. Soc.* **1935**, *57*, 1743–1745.
(57) Perkin, Jr., W. H.; Pope, W. J. *J. Chem. Soc. Trans.* **1908**, *93*, 1075–1077.
(58) Perkin, W. H.; Pope, W. J.; Wallach, O. *J. Chem. Soc. Trans.* **1909**, *95*, 1789–1802. Although this paper does not have Jr. after Perkin, it is clear from papers in 1908 and 1911 that this author is Perkin, Jr. Perkin, Sr., died in 1907.
(59) Everest, A. E. *Chem. News J. Phys. Sci. (London)* **1909**, *100*, 295.
(60) Perkin, Jr., W. H.; Pope, W. J. *J. Chem. Soc. Trans.* **1911**, *99*, 1510–1529.
(61) Everest, A. E. *Proc. Chem. Soc. (London)* **1911**, *27*, 285–290.
(62) Marsh, J. E. *Proc. Chem. Soc. (London)* **1911**, *27*, 317–319.
(63) van't Hoff's "Dix Années"[64] was translated by Marsh.[65] It may seem strange that the translator was obviously unaware of van't Hoff's second case of optical activity. However, as will be seen again (section II.D), there is some inconsistency in van't Hoff's treatments. Although his second case of optical activity is mentioned in "La Chimie",[39,44] as well as in both the first[40,45] and second[66] editions of "Die Lagerung", it is not mentioned in the "Dix Années". Nor is it referred to in "Stéréochimie",[67] a new edition of the "Dix Années", which Meyerhoffer prepared with van't Hoff's help. These omissions have led some who have seen the "Dix Années", but not the "La Chimie", to wonder whether we owe the second case of optical activity to Dr. F. Herrmann, the translator into German of "La Chimie". Perkin and Pope were aware of the second case from Eiloart's translation of the second German edition.[66,68]
(64) van't Hoff, J. H. "Dix Années dans l'Histoire d'une Théorie"; P. M. Bazendijk: Rotterdam, 1887. As the title page shows, this is the second edition of "La Chimie", ref 39.
(65) Marsh, J. E. "Chemistry in Space"; Clarendon Press: Oxford, 1891; p 115.
(66) van't Hoff, J. H. "Die Lagerung der Atome in Raume", 2nd ed.; F. Vieweg und Sohn: Braunschweig, 1894; pp 75–76.
(67) van't Hoff, J. H. "Stéréochimie"; Meyerhoffer, W., Ed.; Georges Carré: Paris, 1892; Part 6, pp 107–121. As shown in the title page, this is the second edition of the "Dix Années".
(68) Eiloart, A. "The Arrangement of Atoms in Space"; Longmans, Green and Co.: London, 1898. A translation *and enlargement* of the second edition of "Die Lagerung".
(69) Marsh, J. E. *Phil. Mag.* **1888**, *26*, 426–434.
(70) Reference 47, pp 864–867, 888.
(71) These last phrases are an almost literal translation of Jaeger's remarks before the Société Chimique in Paris in 1923. They were essentially reaffirmed in his George Fisher Baker lectures at Cornell University in 1929.[72]
(72) Reference 49, pp 40–42, 52–54.
(73) Dr. R. S. Cahn (see ref 78) and Nobelist V. Prelog concur that C-4 is a chiral center: R. S. Cahn, letter of Dec. 11, 1974, to Professor W. Klyne, University of London; Professor V. Prelog, Eidgenössische Technische Hochschule, Zürich, Switzerland, letter of Dec. 13, 1977.
(74) The term "isochronism" to describe the situation in which two nuclei resonate at the same nuclear magnetic resonance frequency was first

used by Abragam.[75] Mislow and Raban ascribe the term "anisochronous" to Dr. G. Binsch.[76]

(75) Abragam, A. "The Principles of Nuclear Magnetism"; Clarendon Press: Oxford, 1961; p 480.
(76) Mislow, K.; Raban, M. In "Topics in Stereochemistry"; Allinger, N. L.; Eliel, E. L., Eds.; Interscience: New York, 1967; Vol. 1, Chapter 1, p 23, footnote. In general, Chapter 1 is one of those "books" which Lord Bacon said must be chewed through. Doing that provides a feast; haste gives indigestion.
(77) Reference 76, pp 19–22.
(78) Cahn, R. S. *J. Chem. Educ.* **1964**, *41*, 116–125, 508.
(79) Reference 39, pp 39–41 and Plate III, Figures XLVIII and XLIX.
(80) Those who have not read the sarcastic denunciation of van't Hoff's ideas by Hermann Kolbe, one of the leading theoreticians of the 19th century,[81] should see ref 38. Kolbe remained unconvinced and repeated his denunciations 4 years later.[82]
(81) Kolbe, Hermann. *J. Prakt. Chem.* **1877**, *15*, 473–477.
(82) Kolbe, Hermann, *J. Prakt. Chem.* **1881**, *23*, 305–323. At p 309 Kolbe continues to rail at the ideas of spatial distribution of atoms, motion of atoms in molecules, and the benzene ring.
(83) Reference 39, p 11.
(84) Reference 40, p 10.
(85) Fischer, Emil. *Chem. Ber.* **1891**, *24*, 1836–1845, at p 1839 and footnote 1.
(86) Reference 40, p 12, footnote.
(87) Landolt, Hans. "Das Optische Drehungsvermögen Organischer Substanzen", 2nd ed.; Braunschweig: F. Vieweg und Sohn, 1898; pp 49–50.
(88) Reference 87, p 50. "Rechnet man dieses mittelste Kohlenstoffatom als asymmetrisch, was es in Wirklichkeit nicht ist (Pseudo-Asymmetrie), so wird *n* ungerade, und es gelten die Formeln. . ." "If the most centrally located carbon atom is considered asymmetric, which in reality it is not (pseudoasymmetry), then *n* becomes odd and formulas . . . are valid."
(89) Reference 87, p 121. "Wenn dagegen das mittelste Kohlenstoffatom ausser mit den beiden symmetrischen Gruppen noch mit zwei unter sich *verschiedenen Radicalen* verbunden ist, so resultirt die Zahl der inactiven Isomeren N_i aus der Formel: $N_i = 2^{(n-1)/2}$, wenn *n* die Anzahl der asymmetrischen Kohlenstoffatome, und zwar das *mittelste Kohlenstoffatom als ein ebensolches angesehen* bedeutet. In Wirlichkeit lässt sich indess dieses letzere nicht als ein asymmetrisches betrachten, weil, wie ein Blick auf die nachfolgenden Configurationsformeln zeigt, durch dasselbe und die damit verbundenen ungleichen Radicale eine Symmetrieebene gelegt werden kann. Schaltet man dieses mittelste Kohlenstoffatom aus, so ist dann die Anzahl der configurationsinactiven Isomeren $N_i = 2^{n/2}$ gegeben." (emphasis in the original).
(90) Reference 40, pp 16–18, is especially clear on this point. The central carbon of $C(R_1R_2R_3R_4)$ is called asymmetric because all symmetry is gone from the molecule. van't Hoff carefully noted that the asymmetry does not reside in the carbon *atom*, but in the two nonsuperposable tetrahedral arrangements which could be postulated.
(91) Werner, Alfred. "Lehrbuch der Stereochemie"; G. Fischer: Jena, 1904; pp 28–29, especially pp 81–83, and 86.
(92) van't Hoff, J. H. "Die Lagerung der Atome im Raume", 3rd ed.; F. Vieweg und Sohn: Braunschweig, 1908; pp 21–22.
(93) Mohr, E. *J. Prakt. Chem.* **1903**, *68*, 369–384.
(94) Leonard, J. E.; Hammond, G. S.; Simmons, H. E. *J. Am. Chem. Soc.* **1975**, *97*, 5052–5054. Hammond and co-workers refer to Mohr's paper as a "classic in stereochemistry". Their paper also gives valuable insights into the counting of isomers in ring compounds, such as cyclohexane, in which there is interconversion of conformers.
(95) IUPAC Rules for the Nomenclature of Organic Chemistry. Section E: Stereochemistry (Recommendations 1974). *Pure Appl. Chem.* **1976**, *45*, 11–30. Rule *E-4.8*.
(96) Reference 76, p 3.
(97) Reference 76, pp 22–30.
(98) Prelog, V.; Helmchen, G. *Helv. Chim. Acta* **1972**, *55*, 2581–2598.
(99) Ladenburg, A. *Chem. Ber.* **1895**, *28*, 1995–1997.
(100) Groth, P. *Chem. Ber.* **1895**, *28*, 2510–2511.
(101) Reference 92, pp 29–30.
(102) The notation σC_2 means: first do a 180° rotation (in the present case it might be most conveniently done in the diketopiperazine ring, which is presumed to be planar in the averaging of its conformations) and then do a reflection perpendicular to the C_2 axis. Here $\sigma C_2 = S_2$, and this improper axis produces the same result as inversion through a center of symmetry, i. $\sigma C_2 = C_2\sigma$ indicates that the order of these operations is immaterial; mathematically they are said to commute. If, when the complex σC_n operation is done, even though the object (molecule) may not have either a C_n or σ element of symmetry (sic), the new configuration is indistinguishable from the original, the object is said to have an improper axis of rotation, a rotation–reflection axis, or a rotation axis of the second kind, S_n. See ref 8a.
(103) Delépine, Marcel. *Bull. Soc. Chim. Fr.* **1925**, *37*, 197–236, at p 222.
(104) McCasland, G. E.; Proskow, S. *J. Am. Chem. Soc.* **1955**, *77*, 4688–4689; *J. Am. Chem. Soc.* **1956**, *78*, 5646–5652.
(105) Some authors say S_4, rather than the more general S_n, because it is the S_n axis of lowest order which does not require the presence simultaneously of either σ or i.
(106) These molecules have a twofold axis, C_2, as their only true element of symmetry. "True" because E (or I), the identity elements, are mathematical, not physical, elements of symmetry. C_1, a 360° rotation, since it produces no new configuration, is likewise not a true symmetry element. A molecule having a C_n axis as its only symmetry element belongs to point group C_n.[7,8a] To distinguish the C_n axis from the C_n point group, the latter is sometimes denoted $\mathcal{C}_n$.

(107) Wynberg, H.; Hekkert, G. L.; Houbiers, J. P. M.; Bosch, H. W. *J. Am. Chem. Soc.* **1965**, *87*, 2635–2639.

(108) Wynberg, H.; Hulshof, L. A. *Tetrahedron* **1974**, *30*, 1775–1782.

(109) Reference 46, pp 161–162.

(110) Bauman, R. P. "Absorption Spectroscopy"; Wiley: New York, 1963; pp 31–45, 202–219, 314–330.

(111) Buckingham, A. D.; Stiles, P. J. *Acc. Chem. Res.* **1974**, *7*, 258–264, and references given there.

(112) Foss, J. G. *J. Chem. Educ.* **1963**, *40*, 592–597.

(113) Mislow, K. *Science* **1954**, *120*, 232–233.

(114) Mislow, K.; Bolstad, R. *J. Am. Chem. Soc.* **1955**, *77*, 6712–6713.

(115) Reference 50, pp 90–93.

(116) Mislow and Raban have pointed out that, because the term "enantiomer" involves a relationship within a set of two stereoisomers, a molecule should properly be described not as an "enantiomer", but as an "enantiomer of" another molecule. The same is true of "diastereomer" and "diastereomer of", although in this second case the set may be larger than two.[117]

(117) Reference 76, p 4, footnote.

(118) Hobden, M. V. *Nature (London)* **1967**, *216*, 678.

(119) Buerger, M. J. "Elementary Crystallography"; Wiley: New York, 1956; pp 184–185.

(120) Reference 32, pp 300–301.

(121) Reference 6, p 337.

(122) Jerephagnon, J.; Chemla, D. S. *J. Chem. Phys.* **1976**, *65*, 1522–1529. For further explication of the tensor methods used in this article, including the description of the physical properties of condensed matter by tensors, see ref 123.

(123) Jerephagnon, J.; Chemla, D.; Bonneville, R. *Adv. Phys.* **1978**, *27*, 609–650.

(124) Gibbs, J. W. *Am. J. Sci.* **1882**, *23*, 460–476. "The Scientific Papers of J. Willard Gibbs"; Bumstead, H. A.; VanName, R. G., Eds.; Longmans, Green: New York, 1906; pp 195–210, especially pp 208–209.

(125) In section II we used only the Schoenflies notation for point group symmetry, because that is most commonly used by chemists and spectroscopists. In section III, referring to crystals, we put in parentheses the Hermann–Maugin (International) crystallographic notation as well.[7,8a,126–128]

(126) Reference 119, pp 59–67.

(127) Reference 7, Chapter 6.

(128) Reference 8a, Appendix I.

(129) Objects belonging to point group S_n, where n is even, *must* have a $C_{n/2}$ axis coincident with the S_n axis.[7,8a] Although the $C_{n/2}$ axis is thus not strictly an independent element, we follow the character tables customarily used by spectroscopists, in which the $C_{n/2}$ axis is included.[7,8a,130]

(130) Wilson, Jr., E. B.; Decius, J. C.; Cross, P. C. "Molecular Vibrations"; McGraw-Hill: New York, 1955; Appendix X.

(131) Fedorov, F. I. *Opt. Spectrosc. (USSR) (Engl. Transl.)* **1959**, *6*, 49–53; *Opt. Spektrosk.* **1959**, *6*, 85–93. In eq 13, H should be replaced by E. The dots in expressions like $\dot{E}$, which means derivative with respect to time, are often very hard to see in the English version. The translation leaves something to be desired. Concerning this article, R. Clark Jones[134] says, "This is a monumental article, in which the author derives for the first time the correct equation of energy in an anisotropic, optically active, nonabsorbing crystal." (By a typographical error, the original has "absorbing".) The remark under ref 76 applies.

(132) Fedorov, F. I. *Opt. Spectrosc. (USSR) (Engl. Transl.)* **1959**, *6*, 237–240; *Opt. Spektrosk.* **1959**, *6*, 377–383.

(133) Bokut, B. V.; Fedorov, F. I. *Opt. Spectrosc. (USSR) (Engl. Transl.)* **1959**, *6*, 342–344; *Opt. Spektrosk.* **1959**, *6*, 537–541.

(134) Jones, R. C. "A Comedy of Errors". In "Benchmark Papers in Optics"; Swindell, William, Ed.; Halsted Press: New York, 1975; Vol. 1, pp 186 and 240–241.

(135) Pockels, F. "Lehrbuch der Kristalloptik"; B. G. Teubner: Leipzig, 1906; Part II, Chapter 1, pp 313–318.

(136) Nye, J. F. "Physical Properties of Crystals"; Clarendon Press: Oxford, 1957; pp 265–266. An excellent book with a good treatment of the representation of the physical properties of crystals by tensors.

(137) Birss, R. R. "Symmetry and Magnetism", 2nd ed.; "Selected Topics in Solid State Physics"; Wohlfarth, E. P., Ed.; North-Holland: Amsterdam, 1966; Vol. 3, pp 13, 71.

(138) Juretschke, H. J. "Crystal Physics, Macroscopic Physics of Anisotropic Solids"; W. A. Benjamin: Reading, MA, 1974; Chapter 10.

(139) Szivessy, G. "Kristalloptik". In "Handbuch der Physik"; Geiger, H. and Scheel, K., Eds.; Springer-Verlag: Berlin, 1928; Vol. 20, pp 804–837. Many references to the early work on optical activity of crystals, often with critical commentary. The entire article is pp 635–904.

(140) Born, M. "Optik, Ein Lehrbuch der elektromagnetischen Lichttheorie"; Springer-Verlag: New York, 1965; Sect. 83, pp 403–420. (This is the second unaltered edition of the original 1933 work.) Unfortunately Section 83, Optical Rotatory Power of Isotropic Bodies, is not in any of the editions of Born, M. and Wolf, E., "Principles of Optics".
The German text has a few errors: (1) On p 414 immediately following eq 2, it is stated that, for a given frequency, the elements of the *optical gyration tensor* are constants of the medium. This error concerning the nature of the optical gyration tensor has recently been discussed by Jones.[134] (2) On p 418, eq 26 should read $\chi = \pi G/\lambda \bar{n}$, instead of $\bar{n}^2$. (3) On p 418, lines 1 and 2 above eq 28 should read: ". . . d.h. muss bei Vertauschung von s_x in $-s_x$ in $-G$ übergehen" (instead of "in G übergehen").

(141) Mathieu, J. P. "Activité Optique Naturelle". In "Handbuch der Physik"; Flügge, S., Ed.; Springer-Verlag: Berlin, 1957; Vol. 28, pp 338–339.

(142) Landau, L. D.; Lifschitz, E. M. "Electrodynamics of Continuous Media"; Addison-Wesley Press: Reading, MA, 1960; pp 337–343.

(143) Ramachandran, G. N.; Ramaseshan, S. "Crystal Optics". In "Handbuch der Physik"; Flügge, S., Ed.; Springer-Verlag: Berlin, 1961; Band XXV/1, pp 83–84.

(144) Reference 136, pp 270–273.

(145) Bloss, F. D. "Crystallography and Crystal Chemistry"; Holt, Rinehart, and Winston: New York, 1971; p 433. This work has an excellent *physical* introduction to tensors.

(146) Sommerfeldt, E. *Phys. Z.* **1906**, *7*, 207–208, 266.

(147) Voigt, W. *Phys. Z.* **1906**, *7*, 267–269.

(148) Reference 139, pp 808–811.

(149) Futama, H.; Pepinsky, R. *J. Phys. Soc. Jpn.* **1962**, *17*, 725.

(150) Hobden, M. V. *Acta Crystallogr., Sect. A* **1968**, *24*, 676–680.

(151) Hobden, M. V. *Nature (London)* **1968**, *220*, 781; *Acta Crystallogr., Sect. A* **1969**, *25*, 633–638. I am much indebted to Professor E. Donald Bloss, of Virginia Polytechnic Institute and State University, for sending me the references to Hobden's work on cadmium gallium sulfide.

(152) Jenkins, F. A.; White, H. E. "Fundamentals of Optics", 3rd ed.; McGraw-Hill: New York, 1957; pp 575–577, 580–581.

(153) Ditchburn, R. W. "Light", 2nd ed.; Interscience: New York 1963; p 488. 3rd ed.; Academic: New York, 1976; pp 468–469.

(154) Stone, J. M. "Radiation and Optics"; McGraw-Hill: New York, 1963; Chapter 17.

(155) Monochromatic radiation is not essential, but use of it avoids effects due to linear dispersion.

(156) Reference 119, pp 162–164.

(157) The value of n_e is a function of the direction of the light ray through the crystal. It varies continuously from n_0 in the limit, as the oblique ray approaches the normal, to its extreme value at 90° from the normal, which is known as n_e. For positive crystals (e.g., quartz), $n_0 < n_e$, but for negative ones (e.g., calcite), $n_0 > n_e$.[158]

(158) Reference 152, p 535.

(159) Reference 136, Chapter 14.

(160) As in the previous experiments, monochromatic radiation is not essential. However, as we there avoided the effects of linear dispersion, here we avoid those due to rotatory dispersion.

(161) These directions of rotation refer to a light ray advancing into the eye of the observer.

(162) Moscowitz, A. In "Optical Rotatory Dispersion"; Djerassi, C., Ed.; McGraw-Hill: New York, 1960; pp 151–160.

(163) Macdonald, J. R.; Brackman, M. K. *Rev. Mod. Phys.* **1956**, *28*, 393–422.

(164) Schellman, J. A. *Chem. Rev.* **1975**, *75*, 323–331.

(165) Because we are considering only nonabsorbing crystals, we need not consider *singular* axes (Ger., ausgezeichnete Achsen). Along these, for *absorbing* crystals, both optically active and optically inactive, only circularly polarized light is transmitted. The singular axes are not the same as the optic axes.[166–168]

(166) Reference 143, pp 69–70, 90–91, 95–96, 106–107, 590.

(167) Born, M.; Wolf, E. "Principles of Optics", 5th ed.; Pergamon Press: New York, 1975; p 718. Same page in third and fourth editions.

(168) Pancharatnam, S. *Proc. Indian Acad. Sci., Sect. A* **1958**, *48*, 227–244. See R. C. Jones for correction of an error concerning the nature of the optical gyration tensor.[134]

(169) Bragg, W. H. *Proc. R. Soc. London, Ser. A* **1913**, *89*, 575–580.

(170) Bragg, Sir William; Gibbs, R. E. *Proc. R. Soc. London, Ser. A* **1925**, *109*, 405–427.

(171) Tatarskii, V. B. *Sov. Phys.-Crystallog. (Engl. Transl.)* **1964**, *9*, 377–379; *Kristallografiya* **1964**, *9*, 451–454.

(172) Kobayashi, J.; Takahashi, T.; Hosokawa, T.; Uesu, Y. *J. Appl. Phys.* **1978**, *49*, 809–815.

(173) Chern, M.-J.; Phillips, R. A. *J. Opt. Soc. Am.* **1970**, *60*, 1230–1232; **1970**, *60*, 1542(A).

(174) Chern, M.-J.; Phillips, R. A. *J. Appl. Phys.* **1972**, *43*, 496–499.

(175) Reference 139, pp 805–806.

(176) Voigt, W. *Akad. Wiss. Göttingen, Nach.: Math.-Phys. Kl.* **1903**, 155–185.

(177) Voigt, W. *Akad. Wiss. Göttingen, Nach.: Math.-Phys. Kl.* **1903**, 186–202, especially 188–190.

(178) Voigt, W. *Ann. Phys. (Leipzig)* **1905**, *18*, 645–694, at 650–653. Footnote 2, p 645, gives 1904 for my ref 177; 1903 is correct.

(179) Chipart, H. "La Théorie Gyrostatique de la Lumiére"; Gauthier-Villars: Paris, 1904; pp 20–22.

(180) Chipart, H. *C. R. Hebd. Seances Acad. Sci.* **1924**, *178*, 995–996.

(181) Voigt, W. "Lehrbuch der Kristallphysik", 2nd ed.; B. G. Teubner: Leipzig, 1928; pp 314–315. Exactly same pages in 1st ed., 1910.

(182) Jones, R. C. *J. Opt. Soc. Am.* **1956**, *46*, 126–131. This paper, together with the seven on the Jones calculus which preceded it, is reprinted in the series, "Benchmark Papers in Optics/1". See ref 134. The paper just cited comprises pp 234–239 of this book.

(183) Melcher, R. L. *Phys. Rev. Lett.* **1973**, *30*, 125–128.

(184) Reference 140, pp 406–408.

(185) Kobayashi, J.; Bouillot, J.; Kinoshita, K. *Phys. Status Solidi B* **1971**, *47*, 619–628.

(186) Yu, C. C.; Barker, R. C. *J. Opt. Soc. Am.* **1975**, *65*, 1168–1169(A). A paper of exactly the same title as the abstract will be published by these authors in *J. Opt. Soc. Am.*: personal communication, Dr. Chen Cheng Yu, IBM Corp., Poughkeepsie, NY, July 18, 1979.

(187) Uesu, Y.; Sorimachi, H.; Kobayashi, J. *Phys. Rev. Lett.* **1979**, *42*, 1427–1430.

(188) Hofrichter, J. H.; Schellman, J. A. *Jerusalem Symp. Quantum Chem. Biochem.* **1972**, *5*, 787–807. I am indebted to Professor Schellman for drawing my attention to this paper and for sending me a reprint of it.

(189) Norden, Bengt, *J. Phys. Chem.* **1977**, *81*, 151–159.

(190) Saeva, F. D.; Olin, G. R. *J. Am. Chem. Soc.* **1977**, *99*, 4848–4850.

(191) Saeva, F. D.; Olin, G. R.; Turner, S. R.; Yanus, J. F.; Sandman, D. J. *Photog. Sci. Eng.* **1978**, *22*, 129–132.
(192) Reinitzer, Friedrich, *Monatsh. Chem. Verw. Anderer Wiss.* **1888**, *9*, 421–441, at 432.
(193) Reference 6, p 346. Quoted by permission of the publisher, Dover Publications, Inc.
(194) Stegemeyer, Horst, *Angew. Chem., Int. Ed. Engl.* **1972**, *11*, 731; *Angew. Chem.* **1972**, *84*, 720–721.
(195) Stegemeyer, H.; Mainusch, K.-J. *Naturwissenschaften* **1971**, *58*, 599–602.
(196) Williams, Richard. *Phys. Rev. Lett.* **1968**, *21*, 342–344.
(197) Williams, Richard. *J. Chem. Phys.* **1969**, *50*, 1324–1332.
(198) de Gennes, P. G. *Solid State Commun.* **1970**, *8*, 213–216.
(199) Meyerhofer, D.; Sussman, A.; Williams, R. *J. Appl. Phys.* **1972**, *43*, 3685–3689.
(200) Kimura, H. *J. Phys. Soc. Jpn.* **1971**, *30*, 1273–1279; **1971**, *31*, 623.
(201) Nakano, H.; Kimura, H. *J. Phys. Soc. Jpn.* **1969**, *27*, 519–535.
(202) Walker, James *J. Chem. Soc.* **1913**, 1127–1143, at 1131–1133.
(203) Le Bel, J. A. *Bull. Soc. Chim. Paris* **1890**, *3*, 788–796. Sementsov gives additional references on this point.
(204) Reference 39, p 13 and Plate II, Figures XXIII and XXIV.
(205) Reference 40, pp 12–14.
(206) Le Bel, J. A. *Bull. Soc. Chim. Paris* **1892**, *7*, 164; **1894**, *11*, 292–295.
(207) Le Bel, J. A. *Bull. Soc. Chim. Paris* **1882**, *37*, 300–302.
(208) Landolt-Börnstein, New Series; Springer-Verlag: Berlin, 1971; III/5b, pp 941 and 1500.
(209) *Struct. Reports*, **1953**, *17*, 724.
(210) "Beilsteins Handbuch der Organischen Chemie", 4th ed.; Springer-Verlag: Berlin, 1921; Vol. III, pp 763–765.
(211) Federlin, W. *Justus Liebigs Ann. Chem.* **1907**, *356*, 251–280, at 268–272.
(212) Rogers, D. *Nature (London)* **1953**, *171*, 929.
(213) Reference 178, pp 651, 660, 668–673.
(214) Reference 137, pp 56–57, 59.
(215) Reference 140, p 418. See ref 140 for note on error in eq 26 in Born.
(216) Reference 140, p 415, eq 7 and 9.
(217) For the meanings of material (matter, property) tensors and nonmaterial (field, physical) tensors, see Birss[218] and Nye.[219]
(218) Reference 137, pp 13–14, 45.
(219) Reference 136, p 91.

(220) Reference 140, p 408.
(221) Reference 154, pp 424, 439–440, 444.
(222) In his first paper, ref 131, Fedorov assumed no necessary connection between α and β. In a later paper[223] it was shown that β is related to the transpose of α.[224]
(223) Bokut, B. V.; Serdyukov, A. N.; Fedorov, F. I. *Sov. Phys.-Crystallog. (Engl. Transl.)* **1971**, *15*, 871–874; *Kristallografiya* **1970**, *15*, 1002–1006.
(224) Bokut, B. V.; Serdyukov, A. N. *Sov. Phys.-JETP (Engl. Transl.)* **1972**, *34*, 962–964; *Zh. Eksp. Teor. Fiz.* **1971**, *61*, 1808–1813.
(225) Fedorov, F. I. *Sov. Phys.-Crystallog. (Engl. Transl.)* **1958**, *3*, 46–52; *Kristallografiya* **1958**, *3*, 49–56, and references there.
(226) Agranovich, V. M.; Ginzburg, V. L. "Spatial Dispersion in Crystal Optics and the Theory of Excitons"; Wiley-Interscience: New York, 1966; pp 102–108, especially pp 107–108.
(227) Fedorov, F. I.; Bokut, B. V.; Konstantinova, A. F. *Sov. Phys.-Crystallog. (Engl. Transl.)* **1963**, *7*, 738–741; *Kristallografiya* **1962**, *7*, 910–915.
(228) Fedorov et al. actually say "the incident wave is linearly polarized in the plane of incidence" (ref 227 at p 740). However, the conditions which they give for the case of the optical axis perpendicular to the plane of incidence, together with their eq 5, with $B = 0$, show that the electric vector must be parallel to the optic axis. These authors are using plane of polarization in the traditional sense in optics, in which it means the plane containing the magnetic vector, the H vector.[229,230]
(229) Reference 167, p 28.
(230) Hardy, A. C.; Perrin, F. H. "The Principles of Optics"; McGraw-Hill: New York, 1932; p 16.
(231) Reference 7, p 148.
(232) Reference 8a, p 37.
(233) Fedorov and co-workers follow the convention of Szivessy and Münster[234] with respect to the direction for right and left polarization. This is the physicists' former convention: the sense of rotation is determined as light travels *away* from the eye. In recent years the chemists' convention—the sense of rotation is determined as light travels *toward* the eye, which happens when a polarimeter is used[161]—has generally prevailed.[6] For this reason, Nye[136,144] uses a convention for the sign of the components of the optical gyration tensor for quartz crystals opposite to that of Szivessy and Münster.
(234) Szivessy, G.; Münster, C. *Ann. Phys. (Leipzig)* **1934**, *20*, 703–736.